

Zero-latency tracking of predictable targets by time-delay systems†

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There are time delays in human motor control systems. But humans can track predictable targets with zero-latency. To help understand this behaviour, we constructed a model that could overcome a time delay and track predictable targets with zero-latency. The model first identifies the frequency, amplitude and waveform of the target; then it synthesizes a signal that compensates for the time delay, thereby producing zero-latency tracking. The technique used by this model should be applicable in the fields of physiological systems modelling, man-machine systems and robotics.

1. Introduction

When a ball rolls in front of a car there is a 200 ms time delay before the driver's foot starts to move from the accelerator pedal to the brake pedal. This 200 ms reaction time is typical of all human movements. However, we have found that humans can track predictable targets with zero-latency. In an effort to explain this anomaly, we have developed a control technique that has general applicability. Our scheme allows a time-delay system to track targets with zero-latency whenever the target movement is continuous, smooth, and predictable. In this paper, we explain our control scheme, present our model for the human eye movement system, and suggest other possible applications.

For this control scheme the target-selective adaptive controller constructs an adaptive signal that depends on the frequency, amplitude and waveform of the target movement, as well as on the time delay and dynamics of the plant. When this adaptive signal is applied to the time-delay system it allows zero-latency tracking and improved dynamic performance. Figure 1 shows this scheme applied to a general state-variable feedback control system with a time delay in the forward path. The system input $r_i(t)$, is composed of two parts: the reference source, $r_s(t)$ and the adaptive signal, $r_a(t)$. When $r_s(t)$ is not a known target waveform, $r_a(t)$ is turned off; $r_i(t)$ then equals $r_s(t)$ and the closed-loop transfer function becomes

$$\frac{Y(s)}{R_i(s)} = \frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)} \quad (1)$$

The $\exp(-s\tau)$ term in the numerator is a pure time delay that remains in spite of the feedback. The similar term in the denominator produces the phase lag that reduces the allowable gain. Of the other symbols, $Y(s)$ represents

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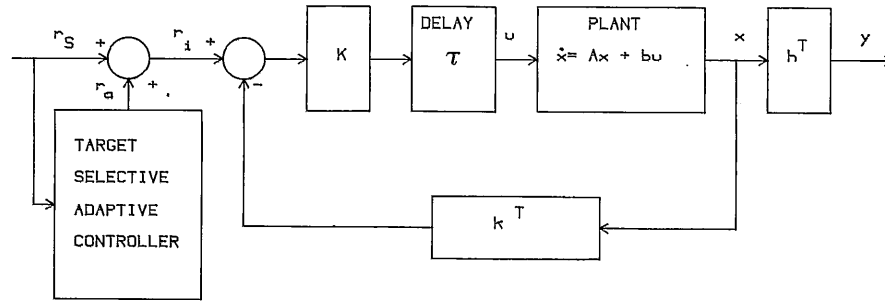


Figure 1. A typical state variable feedback control system with a time delay and a target-selective adaptive controller.

the scalar output, $R_i(s)$ the scalar system input, \mathbf{I} the $n \times n$ identity matrix, \mathbf{A} the $n \times n$ system matrix, K the scalar gain, \mathbf{k}^T the $1 \times n$ feedback control vector, \mathbf{h}^T the $1 \times n$ output coefficient vector, and \mathbf{b} the $n \times 1$ input coefficient vector. Superscript T indicates the transpose operation. The dimensions of the vectors and matrices are such that the numerator and the denominator of (1) are scalars. The feedback vector \mathbf{k}^T and the gain K must be selected to achieve stability.

Many papers have presented control schemes for systems described by eqn. (1). In the following paragraphs, we will review some of this work showing that none of these previous techniques achieves what our present scheme does; namely, produce zero-latency tracking.

A predictor, or an estimator, could be placed in front of such a closed-loop system. However, predicting the future target position would compensate for the effect of the time delay only in the numerator; it would not eliminate the phase lag produced by the presence of the time delay in the denominator.

The method of Smith (1957) and similar more recent algorithms (Deshpande and Ash 1981, Marshall 1979, Watanabe and Ito 1981) improve system performance by removing the effects of the time delay from the denominator of the transfer function, thus allowing larger gains without instability. However, the effects of the time delay remain in the numerator; the system still tracks with a time delay. A Smith predictor in combination with a pure prediction element would allow zero-latency tracking, but such a system would not be physically realizable.

Feedforward control is often used to compensate for the effects of load disturbances (Deshpande and Ash 1981). This technique requires measurement of changes in the load before they have an effect on the controlled output. Our scheme does not consider the effects of load disturbances.

If an appropriate summing block is available, then feedforward (which is closely related to pole cancellation) can be used to create zeros in the transfer function, thereby producing phase advance (D'Azzo and Houpis 1981, McRuer 1980). This technique can compensate for system dynamics, but it does not remove the effects of the time delay from the numerator of eqn. (1), therefore, the system would still track with a time delay.

The method of preview control (MacAdam 1981, Johanness and Govindaraj 1980, Hess 1981), improves the performance of a time-delay system by letting the system track a future value of the target. For man-machine control systems, such as a person driving a car, this is accomplished by providing the driver with a view of the road many seconds ahead of the car. Our target-selective adaptive control scheme does not require display of future target positions. When the entire trajectory is displayed, preview control becomes a subset of optimal control (Johanness and Govindaraj 1980).

Linear quadratic optimal control techniques have been used to model human performance (Kleinman *et al.* 1971, Greene and Ward 1979, McRuer 1980, Kleinman *et al.* 1980). We did not use this approach because these models did not track the target with zero-latency, and the computational complexity made these models physiologically unrealistic.

Kalman filters are often used to predict, or estimate, process states in the presence of gaussian disturbances and gaussian measurement noise (Kalman 1960, Kleinman *et al.* 1971). However, a Kalman filter would be inappropriate for the oculomotor system because observational and disturbance noises are nearly zero. If there were large disturbance or observational noises, then a Kalman filter could be used in conjunction with the target-selective adaptive controller.

Two different inputs can be used for computing the adaptive signal $r_a(t)$. The target-selective adaptive control model of Fig. 1 uses the target motion $r_s(t)$ as an input. Alternatively, the target-selective adaptive controller could use the system output and the system error (the difference between the system output and the system input) to formulate the adaptive signal. This technique will be used later in Fig. 6. Man-machine systems that use this difference signal are called compensatory systems (McRuer 1980).

Conventional adaptive control systems monitor inputs and outputs and then vary gain elements to compensate for changes in system parameters (Landau 1979, Narendra and Monopoli 1980). The adaptive control technique presented here is not of this type. In our scheme, the adaptive controller monitors inputs and outputs, and produces an adaptive signal that is used as an auxiliary input signal. Landau (1974) calls this technique 'signal synthesis adaptive control' and shows that these two adaptive control techniques are mathematically equivalent.

The two major differences between our target-selective adaptive control scheme and these other control techniques are that (i) our scheme is designed for predictable target waveforms and (ii) our scheme produces zero-latency tracking.

2. The target-selective adaptive control scheme

The following sections provide four examples of compensation for plant time delays in systems with predictable inputs. The first compensates for the time delay and plant dynamics; the second compensates for the time delay and provides arbitrary pole placement; the third compensates for the time delay without requiring control gain changes; and the fourth compensates for the time delay while leaving the transient response unchanged.

2.1. Compensation for time delay and system dynamics

In this first example the system output is made identically equal to the reference input: $y(t) = r_s(t)$. The system input, $r_i(t)$, is the sum of $r_s(t)$, the reference source, and $r_a(t)$, the generated adaptive signal. When $r_s(t)$ is not a known predictable target waveform, $r_a(t)$ is turned off. When $r_s(t)$ is a known predictable target waveform, $r_a(t)$ augments $r_s(t)$ to achieve zero-latency tracking.

Applying the requirement $Y(s) = R_s(s)$ to eqn. (1) produces

$$R_s = \left[\frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)} \right] (R_s + R_a) \quad (2)$$

For notational simplicity we omit the function's argument when it is complex frequency, s . Solving for R_a yields

$$R_a = \left[\frac{\exp(s\tau)}{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K} + \frac{\mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}}{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}} - 1 \right] R_s \quad (3)$$

The time delay τ , the matrix \mathbf{A} , and the vectors \mathbf{b} , \mathbf{k}^T , and \mathbf{h}^T must be known. If $r_s(t)$ can be estimated, then $r_a(t)$ can be computed in advance. For this example the output was made equal to the input; we compensated for both the time delay and the plant dynamics.

2.2. Compensation for time delay with pole adjustment

It may not be necessary, or computationally efficient in real-time computer control, to compensate completely for the system dynamics. This second example demonstrates that it is possible to cancel the effects of the time delay and also place the poles at any desired location. Let the desired new forward gain coefficient be K_a and the desired new feedback vector be \mathbf{k}_a^T . Substituting these requirements in eqn. (1) yields

$$Y = \left[\frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K_a}{1 + \mathbf{k}_a^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K_a} \right] R_s \quad (4)$$

$$= \left[\frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)} \right] (R_s + R_a) \quad (5)$$

Solving for R_a yields

$$R_a = \left[\frac{K_a}{K} \exp(s\tau) + \frac{\mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K_a}{1 + \mathbf{k}_a^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K_a} - 1 \right] R_s \quad (6)$$

The first term of the right hand side of eqn. (6) is the relationship of R_a to future values of the reference input. The remaining two terms represent the differential relationship between R_a and current value of R_s .

For a known input reference R_s , one can readily compute R_a . Thus, the system response can be modified to have a desired characteristic response and no time delay.

2.3. Compensation for time delay without gain changes

It may be necessary to cancel the effects of the time delay without inserting new gains; that is $\mathbf{k}_a^T = \mathbf{k}^T$ and $K_a = K$. For this requirement we obtain a simplified case of eqn. (6)

$$R_a = \left[\exp(s\tau) - \frac{1}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K} \right] R_s \quad (7)$$

This form has simple implementation requirements and lends itself easily to real-time computer control. It is used when the closed-loop system time-delay is unacceptable but the system pole locations are not critical.

2.4. Compensation without changes in pole locations

The auxiliary input from eqn. (7) acts not only to cancel the effects of $\exp(-s\tau)$ on the closed-loop system numerator, but it also eliminates the effect of $\exp(-s\tau)$ on the pole locations. To leave the closed-loop poles in the same location as eqn. (1), the system response to known targets may be specified as

$$Y = \left[\frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)} \right] R_s \quad (8)$$

Substituting this requirement into eqn. (1) yields

$$\begin{aligned} Y &= \left[\frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)} \right] R_s \\ &= \left[\frac{\mathbf{h}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)}{1 + \mathbf{k}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K \exp(-s\tau)} \right] (R_s + R_a) \end{aligned} \quad (9)$$

Solving for R_a produces

$$R_a = [\exp(s\tau) - 1] R_s \quad (10)$$

Note that this is not the same result obtained by placing a predictor of $r_s(t + \tau)$ before the summing junction in Fig. 1. Such a predictor would leave the effect of the time delay in the denominator.

Input signal waveforms may be predictable for human tracking of certain visual target waveforms and robotic tracking of objects on a moving platform. Both applications have large signal processing time delays. Observations of human tracking indicate that input adaptation does occur and zero-latency tracking results. Although present robotic visual systems do not use such adaptive techniques, it might be advantageous.

3. Human eye movement velocity servo system

The foregoing formulations require both analytical computation and target prediction. This approach implicitly requires target identification and error evaluation. Computer control systems have the flexibility to accomplish these tasks. Biological systems also exhibit these capabilities, particularly for the control of eye movements (Bahill and McDonald 1983, Eckmiller and Mackeben 1980, Greene and Ward 1979, Young 1977, Stark *et al.* 1962, Dallos and Jones 1963).

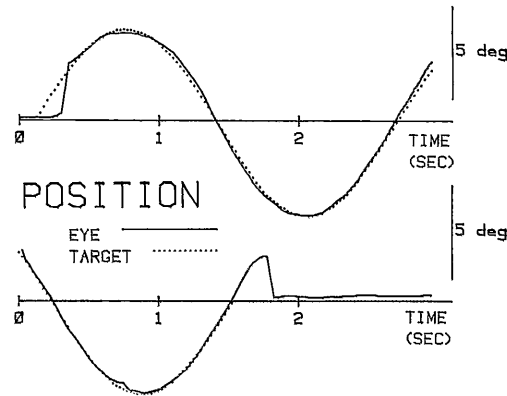


Figure 2. A typical beginning (upper) and ending (lower) of sinusoidal tracking by a normal human. The eye position (solid line) is superimposed on the target position (dotted line). Target movements are $\pm 5^\circ$; the time axis is calibrated in seconds. Mean squared error between the target and eye positions is 0.12 deg^2 for the top and 0.21 deg^2 for the bottom.

The human eye movement control system performs in a manner suggesting target-selective adaptive control. When a target starts moving there is a 150 ms delay before the eye starts moving, as shown in Fig. 2 (upper). When the target stops, the eye continues to follow the predicted target for 150 ms (Fig. 2 lower). However, when a human tracks a predictable target, the brain identifies the target within one half-cycle and generates an adaptive signal, $r_a(t)$, that makes the phase error approach zero (Fig. 3).

This change to zero-latency tracking is a result of control signal changes and not to changes in plant characteristics. The extraocular plant—consisting of the eyeball, the extraocular muscles, the nerve fibres, and the suspensory tissues—cannot change quickly. Neurophysiological studies suggest that

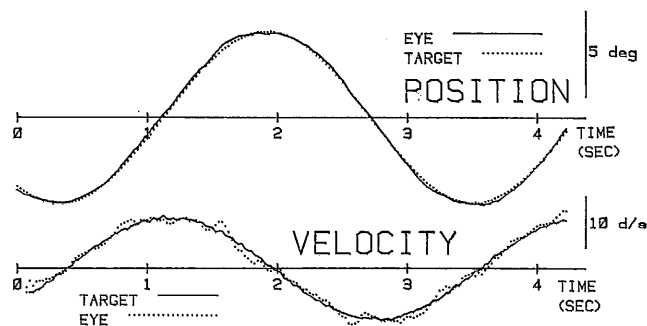


Figure 3. Humans can track sinusoidal targets with zero-latency. The target and the eye positions (top) and the target and eye velocities (bottom) are superimposed. Solid lines are the eye position and the target velocity; dotted lines are the target position and eye velocity. Mean squared position error is 0.02 deg^2 . Mean squared velocity error is $0.7 \text{ deg}^2/\text{sec}^2$. Derivatives were computed with a two point central difference algorithm that low-pass filtered the data at 8.9 Hz.

changes in the plant or controller take hours to occur (Miles and Eighmy 1980). Thus, the rapid performance change is being caused by the brain, presumably by changing the system input, $r_i(t)$.

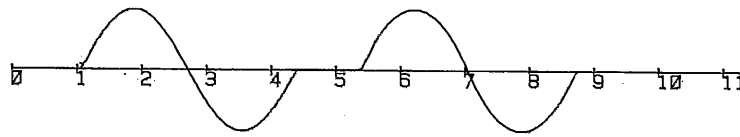


Figure 4. Two smooth, periodic, target waveforms; one sinusoidal and the other based on parabolic segments. The mean squared error between the two waveforms is 0.03 deg^2 .

Similar zero-latency tracking is observed for target waveforms composed of regular parabolic segments. While such a target bears a strong resemblance to a sinusoid, as shown in Fig. 4, the velocities are distinctly different. The data of Fig. 5 show human tracking of sinusoidal and parabolic targets.

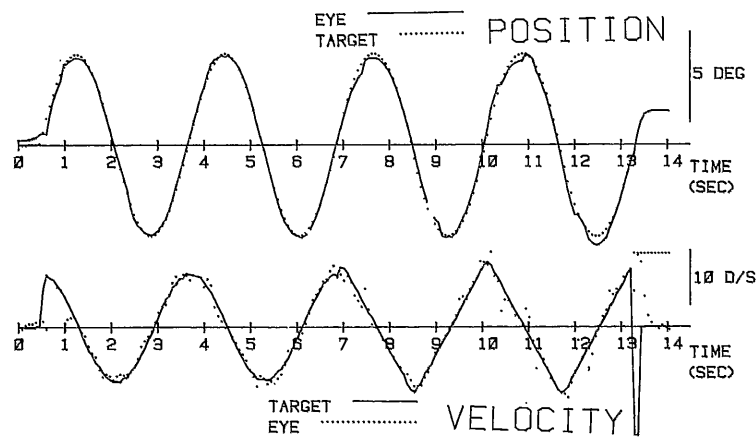


Figure 5. The human can track sinusoidal and parabolic waveforms equally well. For the sinusoidal portion the eye velocity dots cluster around the sinusoidal target velocity line. For the parabolic portion, the eye velocity dots cluster around the linear target velocity line, and at the turn-arounds the eye velocities are larger and the waveform is more pointed. The mean squared errors between target and eye positions were 0.16 deg^2 for the sinusoidal portion, and 0.17 deg^2 for the parabolic portion.

The full model for the eye movement control system is shown in Fig. 6. The smooth pursuit branch of this model acts as a velocity tracking system. The dynamics of the extraocular plant are very fast compared to the dynamics of the smooth pursuit branch (Bahill 1981) and the limiter does not affect the

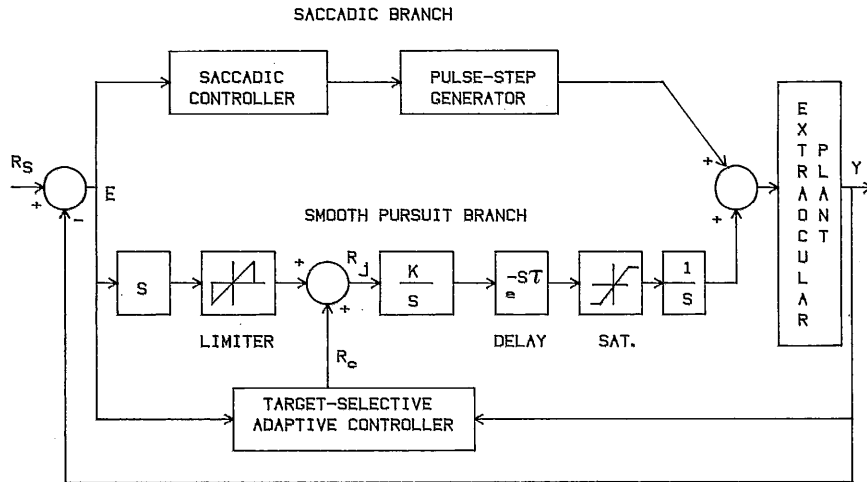


Figure 6. The target-selective adaptive control model for the human eye movement system including saccadic branch, smooth pursuit branch, and adaptive controller.

operation of the adaptive controller. Therefore, for pedagogical purposes, the smooth pursuit system can be modelled with the reduced order model of Fig. 7, which is a velocity tracking system with all dynamics modelled by a single integrator in the forward path.

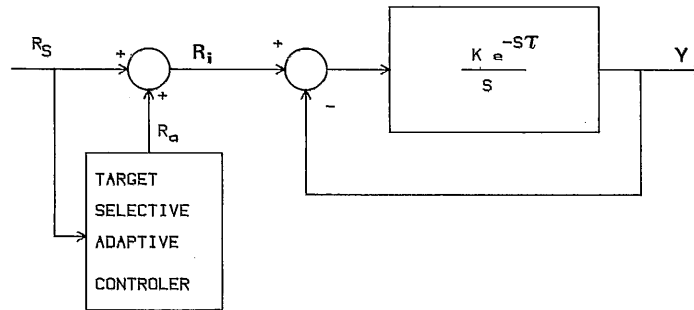


Figure 7. Reduced order model for the human smooth pursuit eye movement system.

The following sections describe two methods for computing the target-selective adaptive signal for this model.

3.1. *Waveform duplication method for computing the adaptive signal*

Within the linear region of closed-loop control, eqn. (4) can be used to determine the correct control for sinusoidal or parabolic targets. For the eye movement system $\mathbf{k}^T = \mathbf{h}^T = 1$, and eqn. (4) simplifies to

$$R_a = \frac{\exp(s\tau)}{(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}K} R_s \tag{11}$$

For the simplified model of Fig. 7, $\mathbf{A} = 0$ and $\mathbf{b} = 1$. Therefore

$$r_a(t) = \frac{1}{K} \dot{r}_s(t + \tau)$$

R_a of Fig. 7 and R_c of Fig. 6 serve the same purpose, but are applied at different places in the models. R_c , called the compensatory signal, is applied after the differentiator in the velocity branch. Therefore, R_c must be the time derivative of R_a . Thus

$$r_c(t) = \frac{1}{K} \frac{d}{dt} \dot{r}_s(t + \tau) \quad (12)$$

The adaptive controller must be able to predict future values of \dot{r}_s and it must be able to compute first derivatives. Then it can produce the compensatory signal r_c that allows the smooth pursuit system to compensate for the time delay. These are reasonable computations for the human brain.

For a sinusoidal target

$$\dot{r}_s(t) = A_s \omega_s \cos(\omega_s t)$$

and the compensatory signal becomes

$$r_c(t) = \frac{-A_s \omega_s^2}{K} \sin(\omega_s(t + \tau)) \quad (13)$$

This compensatory signal, $r_c(t)$, can be computed after the amplitude A_s and frequency ω_s have been determined.

For a parabolic target

$$\left. \begin{aligned} \dot{r}_s(t) &= \frac{-32A_s}{T_s^2} \left[t - \frac{T_s}{4} \right] && \text{for } 0 < t < \frac{T_s}{2} \\ \dot{r}_s(t) &= \frac{+32A_s}{T_s^2} \left[t - \frac{3T_s}{4} \right] && \text{for } \frac{T_s}{2} < t < T_s \\ r_c(t) &= \frac{-32A_s}{KT_s^2} && \text{for } -\tau < t < \frac{T_s}{2} - \tau \\ r_c(t) &= \frac{+32A_s}{KT_s^2} && \text{for } \frac{T_s}{2} - \tau < t < T_s - \tau \end{aligned} \right\} \quad (14)$$

where T_s is the period of the target and A_s is the amplitude.

Typical performance of the full target-selective adaptive control model of Fig. 6, with the adaptive control equations (12)–(14), is shown in Fig. 8. This model performs better than the human subject. It requires the identification of the target waveform (the present menu of waveforms contains square, triangular, sinusoidal, parabolic, and cubic waveforms), the determination of the target amplitude and frequency, and knowledge of the plant time delay and plant dynamics.

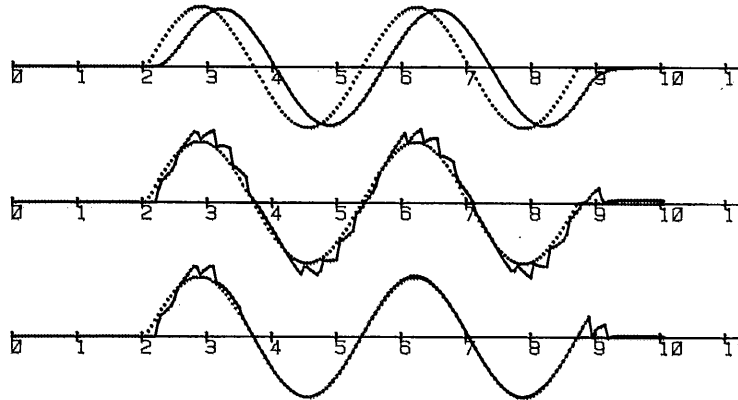


Figure 8. The full model (solid line) tracking a sinusoidal target (dotted line). For the top record the smooth pursuit branch was turned on, but the saccadic branch and the adaptive controller were turned off. For the middle record the smooth pursuit and saccadic branches were turned on, but the adaptive controller was turned off. For the bottom record all three subsystems were turned on. The target movement corresponds to $\pm 5^\circ$ from primary position. The time axis is calibrated in seconds.

3.2. Difference equation method for computing the adaptive signal

The second method of synthesizing the signal, r_c , uses a second-order difference equation. The sampling period of the difference equation is the same as the integration step of the model simulation. The compensatory signal is

$$r_c([n+1]h) = ar_s(nh) - br_s([n-1]h) + cr_s([n-2]h) \quad (15)$$

where

$$a = \frac{\tau}{h^2 K} \quad b = \frac{2\tau - h}{h^2 K} \quad c = \frac{\tau - h}{h^2 K}$$

The element τ is system time delay, h is adaptive sampling period (normally 5 ms), n is the index for discrete time, and K is the system forward path gain. Computer simulation of the model with the $r_c(t)$ of eqn. (15) yielded performance comparable to human subjects for the sinusoidal and parabolic waveforms. If more complex target waveforms were to be tracked, the coefficients of this equation could be adjusted with an adaptive filter (Widrow 1971), or this difference equation could be replaced with an augmented Kalman filter.

4. Robotic position control

Robots using television cameras for visual guidance have requirements analogous to the eye movement control above. There are large time delays for processing camera pictures. The time delays depend on the complexity of the visual environment. For recognition of simple objects, typical time delays exceed one second (Agin 1979). In many applications, objects are carried by conveyer belts and rotating platforms that move with a limited number of predictable trajectories. Identification of the position and orientation of a part should enable the manipulator to apply target-selective adaptive control and acquire the part faster than when only visual feedback is available.

Proportional plus Derivative (PD) control, representative of commonly found controllers, can be used to model the control of a linear manipulator with camera input. The manipulator plant is fast, so the adaptive controller can ignore its dynamics and consider only the dynamics of the compensation branch.

The requirement for adaptation is obtained from eqn. (4) and yields

$$\dot{r}_a(t) = \frac{-K_p}{K_v} r_a(t) + \frac{1}{K_v} r_s(t + \tau) \tag{16}$$

where K_p and K_v are respectively the proportional and derivative gains of the PD controller. One method of generating this adaptive signal is the analogue scheme shown in Fig. 9. A second method of calculating the adaptive signal $r_a(t)$ is illustrated by the following equations.

For sinusoidal targets, the target-selective adaptive control equations are

$$\left. \begin{aligned} r_s(t) &= A_s \sin(\omega t) \\ r_a(t) &= \alpha \sin(\omega(t + \tau) + \phi) \end{aligned} \right\} \tag{17}$$

where

$$\alpha = \frac{A_s K_v}{K_p^2 + \omega^2 K_v^2} \sqrt{1 + \omega^2} \quad \text{and} \quad \tan(\phi) = -\omega$$

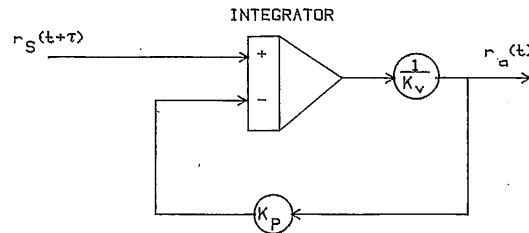


Figure 9. An analogue system for computing the adaptive signal of eqn. (16).

For a target composed of parabolic segments the reference signal and the adaptive signal for the first half period are given by

$$\left. \begin{aligned} r_s(t) &= A_s \left[1 - \left[\frac{t - \frac{T}{4}}{\frac{T}{4}} \right]^2 \right] \quad \text{for } 0 < t < \frac{T}{2} \\ r_a(t) &= \alpha(t + \tau)^2 + \beta(t + \tau) + \gamma \quad \text{for } -\tau < t < \frac{T}{2} - \tau \end{aligned} \right\} \tag{18}$$

where

$$\alpha = \frac{-A_s}{K_p \left[\frac{T}{4} \right]^2}$$

$$\beta = \frac{2A_s}{K_p \left[\frac{T}{4} \right]} \left[1 + \left[\frac{K_v}{K_p \left[\frac{T}{4} \right]} \right] \right]$$

$$\gamma = - \left[\frac{K_v}{K_p} \right] \beta$$

The equation for the last half-cycle has a minus sign instead of a plus sign. These target-selective adaptive control equations, (16)–(18), produce zero-latency tracking for a typical physical plant, a PD controller, and a time delay.

5. Conclusions

A system with a target-selective adaptive controller would be stable for any stable closed-loop system. Combinations of the bounded input are being applied to a stable closed-loop feedback system so the resulting system must be stable. The criterion for turning the adaptive signal, $r_a(t)$, on and off is mean squared error between the reference signal and the system response. Any contrived input that caused the target-selection mechanism to fail would simply increase the squared error and turn off the adaptive process, causing the system to behave as the original, stable, feedback control system. On the other hand, many physiological systems are actually unstable! It is only the continuous action of an intelligent controller that keeps them under control. One example of such an intelligent controller is the target-selective adaptive controller of the smooth pursuit eye movement system.

Our target-selective adaptive control scheme is not restricted to closed-loop systems described by linear state variable feedback as shown in Fig. 1. For new system designs this may be the most convenient representation. The feedback vector \mathbf{k}^T would be known because it would have been selected to satisfy some performance criteria. However, for many existing physical and biological systems \mathbf{k}^T would not be known and would be difficult to compute. In these cases, the closed-loop system would not be modelled with state variable feedback techniques. Instead, suitable experiments would be performed to derive an appropriate transfer function that would describe the plant. This transfer function would then be used to compute the adaptive signal, r_a . The two examples presented in this paper used such simple transfer functions.

There are two important sensitivity functions for most models; the sensitivity to load disturbances and the sensitivity to incorrect parameter estimates. The sensitivity to load disturbances is independent of the target-selective adaptive controller; it is determined by the closed-loop non-linear transfer function of the time-invariant system. The sensitivity to the target-selective adaptive controller's errors in estimating the target parameters (for example, frequency, amplitude or waveform) and the extra-ocular plant parameters has been calculated. Such errors cause the model to track like a

tired or confused human ; position mean squared errors were of the order of one squared degree.

Most control systems are designed without consideration of specific target or input waveforms. However, in systems and models with time delays it may be advantageous to include additional control that will allow compensation for known time delays while tracking certain known predictable targets. The target-selective adaptive control technique developed in this paper demonstrates such controllers.

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REFERENCES

- AGIN, G., 1979, *Proceedings of 9th International Symposium on Industrial Robots*, Washington D.C.
- BAHILL, A. T., 1981, *Bioengineering : Biomedical, Medical and Clinical Engineering* (Englewood Cliffs, New Jersey : Prentice-Hall).
- BAHILL, A. T., and McDONALD, J. D., 1983, *Vision Res.*, **23** (in the press).
- D'AZZO, J. J., and HOUPIS, C. H., 1981, *Linear Control System Analysis and Design* (New York : McGraw-Hill Company).
- DALLOS, P. J., and JONES, R. W., 1963, *I.E.E.E. Trans. autom. Control*, **8**, 218.
- DESHPANDE, P. B., and ASH, R. H., 1981, *Elements of Computer Process Control With Advanced Control Applications* (Research Triangle Park, North Carolina : Instrument Society of America).
- ECKMILLER, R., and MACKEBEN, M., 1980, *Brain Res.*, **184**, 210.
- GREENE, D. F., and WARD, F. E., 1979, *Biol. Cyber.*, **33**, 1.
- HESS, R. A., 1981, *I.E.E.E. Trans. Syst. Man Cyber.*, **11**, 262.
- JOHANSEN, G., and GOVINDARAJ, T., 1980, *I.E.E.E. Trans. Syst. Man Cyber.*, **10**, 249.
- KALMAN, R. E., 1960, *Trans. ASME J. Basic Engr. Series D.*, **82**, 35.
- KLEINMAN, D. L., BARON, S., and LEVISON, W. H., 1971, *I.E.E.E. Trans. autom. Control.*, **16**, 824.
- KLEINMAN, D. L., PATTIPATI, K. R., and EPHRATH, A. R., 1980, *I.E.E.E. Trans. Syst. Man Cyber.*, **10**, 624.
- LANDAU, Y. D., 1974, *Automatica*, **10**, 353 ; 1979, *Adaptive Control* (New York : Marcel Dekker).
- MACADAM, C. C., 1981, *I.E.E.E. Trans. Syst. Man Cyber.*, **11**, 393.
- MARSHALL, J. E., 1979, *Control of Time-Delay Systems* (Stevenage, United Kingdom : Peter Peregrinus Ltd.).
- MCRUER, D. T., 1980, *Automatica*, **16**, 237.
- MILES, F. A., and EIGHMY, B. B., 1980, *J. Neurophysiol.*, **43**, 1406.
- NARENDRA, K. S., and MONOPOLI, R. V., 1980, *Applications of Adaptive Control* (New York : Academic Press).
- SMITH, O. J. M., 1957, *Chem. Engng. Prog.*, **53**, 217.
- STARK, L., VOSSIUS, G., and YOUNG, L. R., 1962, *I.R.E. Trans. Human Factors Electron.*, **3**, 52.
- WATANABE, K., and ITO, M., 1981, *I.E.E.E. Trans. autom. Control*, **26**, 1261.
- WIDROW, B., 1971, *Aspects of Network and System Theory* (New York : Holt Rinehart and Winston), p. 563.
- YOUNG, L., 1977, *Control of Gaze by Brain Stem Neurons* (Amsterdam : Elsevier/North-Holland Biomedical Press), p. 29.