3146)/1-38/(1977).
Wieslander, J. (1979a): Interaction in computer aided analysis and design of control systems. PhD thesis, Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden, Report CODEN: LUTFD2/(TFRT-1019)/1-222/(1979).
Weislander, J. (1979b): Design principles for computer aided design software. Preprints, IFAC Symposium on CAD of Control Systems, Zurich, 493.
Wieslander, J. (1980a): Interactive programs—General guide. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden, Report CODEN: LUTFD2/(TFRT-3156)/1-30/(1980).
Wieslander, J. (1980b): IDPAC commands—User's guide. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden, Report CODEN: LUTFD2/(TFRT-3157)/1-108/(1980).
Wieslander, J. (1980c): MODPAC commands—User's guide. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden. Report CODEN: LUTFD2/(TFRT-3158)/1-81/(1980).
Wieslander, J. (1980d): SYNPAC commands—User's guide. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden, Report CODEN: LUTFD2/(TFRT-3159)/1-130/(1980).
Wieslander, J. and Elmqvist, H. (1978): INTRAC, A communication module for interactive programs. Language manual. Dept of Automatic Control, Lund Institute of Technology, Lund, Sweden, Report CODEN: LUTFD2/(TFRT-3149)/1-60/(1978).
Wilkinson, J. H. and Reinsch, C. (1971): *Linear Algebra*. Springer-Verlag, Berlin.
Winston, P. H. and Horn, B. K. P. (1981): LISP. Addison-Wesley, Reading, Mass.

Karl Johan Astrom was born in Ostersund, Sweden on August 5, 1934. He was educated at the Royal Institute of Technology (KTH) in Stockholm, Sweden. He has held various teaching positions at KTH, and he has worked for the Research Institute of Swedish Defense, and for IBM. In 1965 he was appointed to the chair of Automatic Control at Lund Institute of Technology (LTH). His main research interests are stochastic control theory, identification, adaptive control, and computer aided design of control systems. Astrom has also worked with industrial applications of automatic control in the fields of inertial guidance, paper mills, flight control, and ship steering. Apart from his professional work he and his family enjoy skiing and sailing.

A Simple Adaptive Smith-Predictor for Controlling Time-Delay Systems

A Tutorial, by A. Terry Bahill

Biomedical Engineering Program, Department of Electrical Engineering, Carnegie-Mellon University, Pittsburgh, PA 15213

ABSTRACT: This heuristic paper presents several simple techniques for analyzing the stability of time-delay systems. It explains the Smith predictor control scheme for time-delay systems and shows how errors in modeling the plant parameters can cause instability. Then an adaptive controller is added to the Smith predictor system; this pedagogical example offers a complete derivation of a simple adaptive control system. Finally, a new control scheme is discussed that allows zero-latency tracking of predictable targets by a time-delay system.

Introduction

If a time delay is introduced into a well tuned system, the gain must be reduced to maintain stability [1]. The Smith predictor control scheme can help overcome this limitation and allow larger gains [2], but it is critical that the model parameters exactly match the plant parameters [3-5]. An adaptive control system [6] can be added to the Smith predictor to change the model parameters, so that they continually match the changing plant parameters [3]. This new system has good performance characteristics, but it tracks input signals with a time delay. In some circumstances it is possible to design time-delay systems that track predictable targets with no latency [7], [8].

The examples of this paper treat time-delay systems, the Smith predictor, and an adaptive control system. The examples are complete and the derivations are explicit; no steps are omitted. Many research papers discuss adaptive control systems, but most of them are too complicated for the novice to understand; few textbooks have incorporated simple examples of adaptive control systems. One purpose of this paper is to fill this gap. This paper shows some simple techniques that can be used to gain insight about time-delay systems, explains the Smith predictor control scheme, and presents a complete, but simple, example of an adaptive control system.

Why are time-delay systems more complicated?

Time delays occur frequently in chemical, biological, mechanical, and electronic systems. They are associated with travel times (as of fluids in a chemical process, hormones in the blood stream, shock waves in the earth, or electromagnetic radiation in space), or with computation times (such as those required for making a chemical composition analysis, cortical processing of a visual image, analyzing a TV picture by a robot, or evaluating the output of a digital control algorithm) [1], [3], [7–10]. Most elementary control theory textbooks slight time-delay systems, because they are more difficult to analyze and design. For example, in time-delay systems initial conditions must be specified for the whole interval from $-\theta$ to 0, where θ is the time delay. For simplicity, in this paper I only discuss steady-state behavior, or equivalently I assume the initial conditions are zero.

A unity-feedback, closed-loop control system with $KGH = K/(\tau s+1)$ has a transfer function of

$$\frac{Y(s)}{R(s)} = \frac{K}{\tau s + 1 + K}$$

This is stable for $-1 < K$. If a time delay of the form $e^{-s\theta}$ is introduced in the forward path, stability is no longer guaranteed. The transfer function of such a system is

$$\frac{Y(s)}{R(s)} = \frac{Ke^{-s\theta}}{(\tau s + 1 + Ke^{-s\theta})} \qquad (1)$$

The stability limits are not obvious. The exponential in the numerator does not

Received June 22, 1982; revised October 25, 1982; revised January 18, 1983. Accepted in revised form by Technical Associate Editor F. Aminzadeh.

0272-1708/83/0500-0016$1.00©1983 IEEE

bother us, therefore, it will be left undisturbed. The exponential in the denominator will be approximated by an algebraic expression. The following four approximation techniques have been suggested.

1. By a mathematician, the Taylor series expansion:

$$e^{-s\theta} = 1 - s\theta + \frac{(s\theta)^2}{2!} - \frac{(s\theta)^3}{3!} + \cdots \tag{2}$$

2. By a process control engineer, the Padé approximation:

$$e^{-s\theta} = \frac{1 - s\theta/2}{1 + s\theta/2} \tag{3}$$

3. By a digital control engineer, the z-transform equivalent:

$$e^{-s\theta} = z^{-nh}$$

where h is the system sampling period, n is an integer, and $nh = \theta$.

4. And by a classical control engineer:

$$e^{-s\theta} = \frac{1}{(1 + s\theta/n)^n}$$

where n is a large number.

The first technique implies that the original system has an infinite number of poles that can be reduced by using an approximation. If θ is small so that $s\theta << 1$ for frequencies of interest, then we can derive a single pole solution, by using $e^{-s\theta} = 1 - s\theta$, in equation (1). The transfer function becomes

$$\frac{Y(s)}{R(s)} = \frac{Ke^{-s\theta}}{(\tau - K\theta)s + (K+1)}$$

By the Routh-Hurwitz criterion this transfer function is stable if all the denominator coefficients are positive. This implies $-1 < K < \tau/\theta$. The following two-pole solution can be derived by substituting the first three terms of (2) into (1):

$$\frac{Y(s)}{R(s)} = \frac{2Ke^{-s\theta}}{K\theta^2 s^2 + 2(\tau - K\theta)s + 2(K+1)}$$

All denominator coefficients will be positive and the system will be stable if

$$0 < K < \tau/\theta.$$

Using the Pade approximation in equation (1) produces a third transfer function.

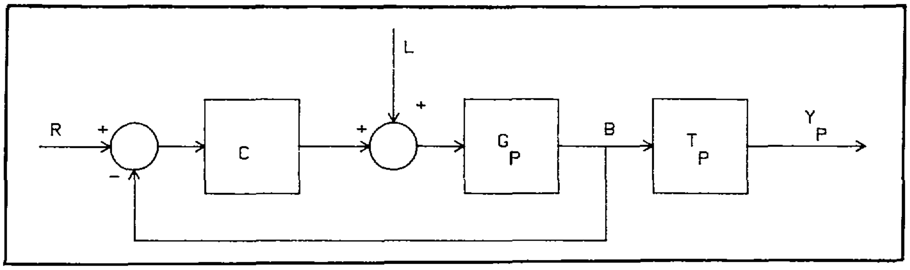


Fig. 1. Block diagram for a typical time-delay system.

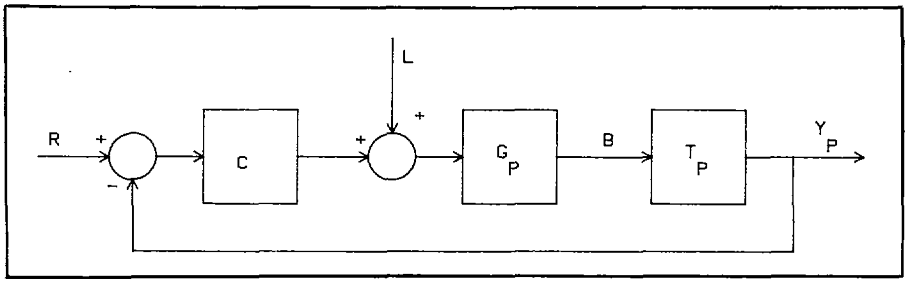


Fig. 2. System performance could be improved if the fictitious variable B could be fed back instead of the output Y.

$$\frac{Y(s)}{R(s)} = \frac{K(2 - s\theta)}{\theta\tau s^2 + (2\tau + \theta - K\theta)s + 2(K+1)}$$

All denominator coefficients will be positive and the system will be stable if $-1 < K < 1 + 2\tau/\theta$. These three approximations yield different stability limits. This should be expected because they are approximations. For the sake of comparison, we can use the Nyquist criterion [11] to derive stability constraints for equation (1). These constraints depend on the relationship of θ to τ:

for $\theta << \tau$ stability requires

$$-1 < K < \sqrt{1 + (1.57\tau/\theta)^2};$$

for $\theta = \tau$ stability requires

$$-1 < K < \sqrt{1 + (2\tau/\theta)^2};$$

and for $\theta >> \tau$ stability requires

$$-1 < K < \sqrt{1 + (\pi\tau/\theta)^2}.$$

The Taylor series approximations err on the safe side. The Padé approximation could indicate stability for some unstable systems.

The introduction of a time delay makes it more difficult to access the stability of a system. The approximation methods shown here are not, in general, good methods for assessing the stability of a system. Sometimes they yield bizzare results, as shown on p. 33 of [3]. However, they do allow us to make the following generalization. Large gains can only be used in time-delay systems if the plant time delay, θ, is small compared to the plant time constant, τ.

The Smith Predictor

If a time delay were introduced into an optimally tuned system, the gain would have to be reduced to maintain stability. The Smith predictor algorithm [2] avoids this reduction of gain and consequent poorer performance.

The following development of the Smith predictor algorithm is based on Despande and Ash [1]. The block diagram for conventional control of a time-delay system is shown in Fig. 1. For simplicity, I will use the shorthand notation of Marshall [3]: R represents the system input, R(s); C represents the controller, C(s); L represents potential load disturbances, L(s); G_p represents plant dynamics, $G_p(s)$; T_p represents the plant time delay, $T_p(s)$; and Y_p represents the plant output, $Y_p(s)$. For a simple first-order plant with a pure time delay $G_p = K_p/(\tau_p s + 1)$ and $T_p = e^{-s\theta_p}$.

As shown in Fig. 1, the process can be conceptually split into delay free system dynamics and a pure time delay. If the ficticious variable B could be measured, we could connect it to the controller, as shown in Fig. 2. This would move the time delay outside the control loop. The signal Y_p would be the same as the signal B after a delay of θ_p. Since there would be no delay in the feedback signal, the response of the system would be improved. Of course, this cannot be done in a physical system, because the time delay is probably distributed—not lumped—and there is no *a priori* reason to place the time delay after the plant dynamics rather than before it.

To improve the design let us model the plant as shown in Fig. 3. G_m represents the model of the plant dynamics, T_m represents the model of the plant time delay, and E represents the error between the output of the model and the output of the plant. For the previous example of a first order process $G_m = K_m/(\tau_m s + 1)$ and $T_m = e^{-s\theta_m}$ Although the fictitious variable B is unavailable, B_m can be used as the feedback signal, as shown in Fig. 3. This arrangement controls the model well, but not the overall system. The control of the system output, Y_p, is open loop: it will not accommodate either load disturbances or inaccurate models. To compensate for these errors a second feedback loop is implemented using E, as shown in Fig. 4. This is the Smith predictor control strategy. The controller C is a conventional PI (Proportional plus Integral), PD (Proportional plus Derivative), or PID (Proportional plus Integral plus Derivative) controller, which can be tuned more closely because the effect of the time delay in the feedback loop has been minimized.

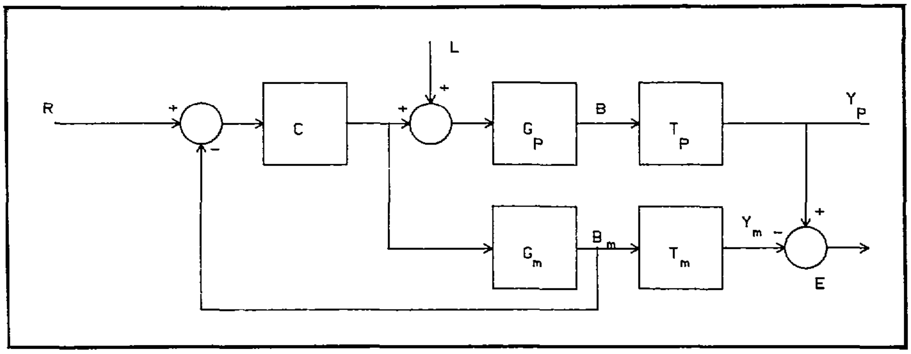


Fig. 3. Preliminary form of the Smith predictor.

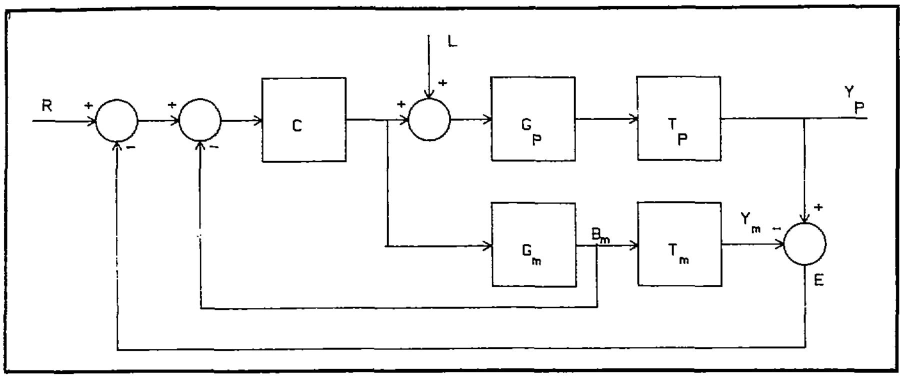


Fig. 4. The complete Smith predictor control scheme.

Sometimes the Smith predictor is drawn as shown in Fig. 5, which is equivalent to Fig. 4. The closed-loop transfer function of this system, for L = 0, is

$$\frac{Y_p(s)}{R(s)} = \frac{CG_pT_p}{1 + CG_m - CG_mT_m + CG_pT_p} \tag{4}$$

If $G_m = G_p$ and $T_m = T_p$ this reduces to

$$\frac{Y_p(s)}{R(s)} = \frac{CG_pT_p}{1 + CG_m} \tag{5}$$

The effects of the time delay have been removed from the denominator of the transfer function, and the system performance has been improved. However, it tracks input variations with a time delay.

Assuming perfect model matching, the transfer function between load disturbance and system output becomes

$$\frac{Y_p(s)}{L(s)} = \frac{G_pT_p[1 + G_mC(1 - T_m)]}{1 + CG_m} \tag{6}$$

Once again there is no time delay in the denominator. However, the system tracks disturbances with a time delay. The system has poor dynamic response unless load disturbances are restricted to frequencies below $2/\theta_p$ Hz, where θ_p is the magnitude of the plant time delay in seconds. For simplicity, we will not consider load disturbances again in this paper.

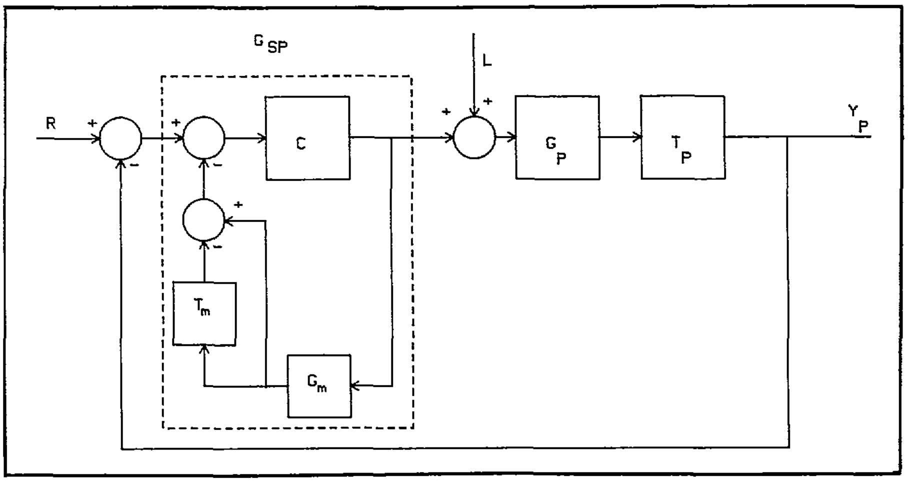


Fig. 5. A rearrangement of the Smith predictor control scheme. Based on [1].

Stability of the Smith Predictor Control System

The dashed box labeled G_{sp} in Fig. 5, called the Smith predictor controller, is a stable closed-loop feedback control system with a transfer function of

$$G_{sp} = \frac{C}{1 + CG_m(1 - T_m)} \tag{7}$$

The condition for instability is $CG_m - CG_mT_m = -1$, or

$$T_m = e^{-s\theta_m} = \frac{1 + CG_m}{CG_m}$$

Clearly the magnitude of the righthand side is never one. Therefore, the controller G_{sp} is stable. However, when this controller is used in a closed-loop system, the result may be unstable.

What happens if the model does not match the plant exactly? Following the development of Marshall [3], let us set

$$G_m = G_p + \Delta G_p = G + \Delta G$$

$$T_m = e^{-s(\theta_p + \Delta\theta_p)} = e^{-s\theta_p} e^{-s\Delta\theta_p}$$

$$= T_p \Delta T_p = T\Delta T$$

This simplified notation gets rid of the subscripts and allows us to redraw Fig. 5 as Fig. 6.

Suppose that the model of the plant dynamics is in error, so that ΔG is non zero, but the model of the plant time delay is correct, so that $T_m = T_p$ and $\Delta T = 1$. The magnitude of the time delay affects the controller via the ΔG term in Fig. 6. This is a positive feedback loop of $T\ \Delta G$. Therefore, errors in estimating the plant gain or time constants could cause instabilities.

If there is an error in modeling only the time delay so that $\Delta G = 0$, then the inner positive feedback loop of Fig. 6 becomes zero. We are left with a simple closed-loop feedback control system with feedback of $(1 - \Delta T)$. The time delay in this feedback loop could cause instability.

Alternatively, we can study the transfer function, equation (4), to see the effects of a mismatch between model and plant parameters. Let us use a simple plant, $G_m = G_p = 1/(s + 1)$, a PD controller, $C = 4(0.5s + 1)$, a plant time delay of 1 sec, and a model time delay of 0.1 sec.

$$= \frac{(2s+4)T_p}{3s + 5 + (2s+4)(T_p - T_m)}$$

Using the Padé approximation for the exponentials yields

$$T_p - T_m = e^{-s} - e^{-0.1s}$$

$$= \frac{-36s}{s^2 + 22s + 40}$$

This produces

$$\frac{Y_p(s)}{R(s)} = \frac{2T_p(s^3 + 24s^2 + 84s + 80)}{3s^3 - s^2 + 86s + 200}$$

The negative coefficient in the denominator shows that this is unstable. The Padé approximation was used here for pedagogical reasons. It is not a good general technique for analyzing stability of Smith predictor systems. Its use indicates stability for many unstable systems. The techniques of Ioannides, Rogers and Latham [4] or Palmor [5] are much more comprehensive. For example, if we use the Padé approximation on equation (4), with $G_p = 1/(s+1)$, $G_m = 1/(3s + 1)$, $C = 4(0.5s + 1)$, and plant and model time constants of 1 sec, we get

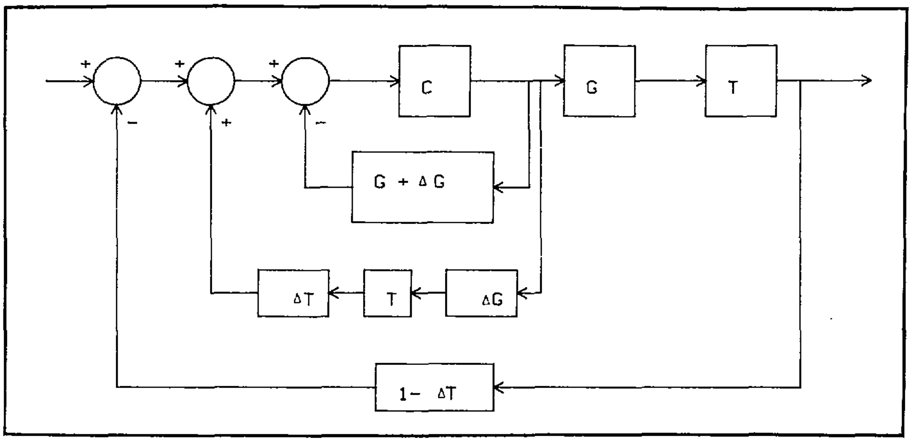


Fig. 6. A rearrangement of the Smith predictor illustrating effects of model and plant mismatches. Based on [3].

$$\frac{Y_p(s)}{R(s)} = \frac{(6s^2 + 14s + 4)T_p}{s^2 + 18s + 5}$$

This transfer function appears to be stable by use of the Padé approximation and the Routh-Hurwitz criterion, although it has been shown to be unstable by the techniques of [4], [5].

An Adaptive Smith Predictor Control System

If is often advantageous to change the controller to compensate for changes in plant parameters caused by age, wear, temperature, fatigue, disease, ocean currents, etc. Let us try to do this for the Smith predictor of Fig. 4. Let us presume that the plant time delay changes, and design a system that will automatically change the model time delay.

The easiest way to do this would be to apply a step input and measure the output. From this we could compute the plant time delay. Normally we cannot do this because applying a step input to an operating plant would disturb the process we wanted to control (this would become annoying if we did it every second). Therefore, we desire a generalized on-line method of adapting the model time delay.

There are many different types of adaptive control schemes [12]. One easily understood scheme is the model reference adaptive control scheme discussed by Landau [6]. In one form of this scheme the controlled system, which is composed of a time-varying plant with adjustable parameters, is in parallel with a well defined model. The parameters of the controlled system are varied to make the controlled system behave like the parallel model [13], [14]. This description is close enough to that of the Smith predictor to make this type of adaptive control applicable.

Two major items must be chosen before an adaptive controller can be designed: a performance criterion and a method of function minimization. Typical performance criteria are minimization of time, energy, monetary cost, error, or square error. Typical function minimization techniques include gradient, conjugate gradient, and Fletcher-Powell.

The adaptive control scheme

Combining examples by Landau [6] and Marshall [3], let us minimize the square error between the model output and plant output, $e = (y_p - y_m)$, using the gradient method of minimization.

The performance function is

$$J = 0.5 \int e^2\ dt.$$

Let the magnitude of the plant time delay be represented by θ_p and assume its initial value, θ_{p0}, has been changed by a small amount called $\Delta\theta_p$. In the model reference approach we would now change gains in the controlled system to minimize the performance function. However, for our Smith predictor we wish to modify the model time delay so that it tracks the plant time delay thus minimizing the performance function. The gradient method tells us $\Delta\theta_m = -k\nabla J$. The gain k must be selected for each application. With only one parameter the gradient is simply the partial derivative.

$$\Delta\theta_m = -k \frac{\partial J}{\partial \theta_m} \quad (8)$$

The mathematics become complicated if we allow both θ_p and θ_m to change continuously. Therefore, although θ_p changes continuously, we will change θ_m only at discrete intervals. We will let the identification and control phases alternate. First we identify changes in θ_p.

$$\Delta\theta_p = -k \frac{\partial J}{\partial \theta_p} \tag{9}$$

The rate at which plant time delay changes is

$$\dot{\theta}_p = \frac{d\theta_p}{dt} = \frac{d}{dt}(\theta_{p0} + \Delta\theta_p)$$

$$= \frac{d}{dt}\Delta\theta_p$$

Using Equation (9)

$$\dot{\theta}_p = -k \frac{d}{dt}\frac{\partial J}{\partial \theta_p}$$

If the adaptation is slow we can interchange the order of differentiation

$$\dot{\theta}_p = -k \frac{\partial}{\partial \theta_p}\frac{dJ}{dt}$$

$$= -k \frac{\partial}{\partial \theta}\frac{d}{dt}\left(0.5\int e^2\, dt\right)$$

$$\dot{\theta}_p = \frac{-k}{2}\frac{\partial e^2}{\partial \theta_p}$$

By the chain rule

$$\dot{\theta}_p = -ke \frac{\partial e}{\partial \theta_p}$$

Finally,

$$\Delta\theta_p = \int \dot{\theta}_p\, dt \tag{10}$$

Now, we can compute the desired change in the model time delay as

$$\Delta\theta_m = \Delta\theta_p = \int \dot{\theta}_p\, dt$$

$$= -k\int e \frac{\partial e}{\partial \theta_p}\, dt \tag{11}$$

But now how do we get $\partial e/\partial\theta_p$? To a first approximation y_m is not a function of θ_p. (For each specific application this should be verified by simulation as was done by Marshall [3].) Therefore,

$$\frac{\partial e}{\partial \theta_p} = \frac{\partial y_p}{\partial \theta_p} \tag{12}$$

Substituting (12) into (11) we get

$$\Delta\theta_m = -k\int e \frac{\partial y_p}{\partial \theta_p}\, dt \tag{13}$$

We are pleased with this result because the $\partial y_p/\partial\theta_p$, called a sensitivity function, can easily be computed. From equation (4) we have

$$M(s) = \frac{Y_p(s)}{R(s)}$$

$$= \frac{CG_pT_p}{1 + CG_m - CG_mT_m + CG_pT_p} \tag{14}$$

For simplicity the transfer function has been labeled M(s), and the complex frequency argument, s, will now be omitted. Now, we calculate the partial derivative

$$\frac{\partial Y_p}{\partial \theta_p}$$

$$= \frac{(-sCG_pT_p)(1 + CG_m - CG_mT_m)R}{(1 + CG_m + CG_pT_p - CG_mT_m)^2}$$

Using (14) twice yields

$$\frac{\partial Y_p}{\partial \theta_p} = -sM(1 - M)R$$

Figure 7 shows one method for producing $\partial y_p/\partial\theta_p$. It has been assumed that the Laplace transform of $\partial y_p(t)/\partial\theta_p$ is equal to $\partial Y_p(s)/\partial\theta_p$.

The pure derivative function shown in this figure would not be physically realizable with analog components. However, with a digital computer it could be approximated over any given frequency range. The boxes labeled G_p and T_p in the lower Smith predictor are supposed to contain the plant values, but they are not available. Therefore, we will have to use G_m and T_m and update them as often as possible.

Computing the gain k

Now, we still have to evaluate the constant k in (13). From equations (11) and (13) we have

$$\Delta\theta_p = -k\int_{t_1}^{t_2} e \frac{\partial y_p}{\partial \theta_p}\, dt \tag{15}$$

The error, e, is a function of the plant time delay. For a small change in the time delay, $\Delta\theta_p$, using the definition of the differential and assuming the initial value of this error is zero (so that $\Delta e = e$), we get

$$e = \Delta\theta_p \frac{\partial e}{\partial \theta_p}$$

Substituting this and (12) into (15) yields

$$\Delta\theta_p = -k\int_{t_1}^{t_2} \Delta\theta_p \frac{\partial y_p}{\partial \theta_p}\frac{\partial y_p}{\partial \theta_p}\, dt$$

$\Delta\theta_p$ can be removed from the integral and

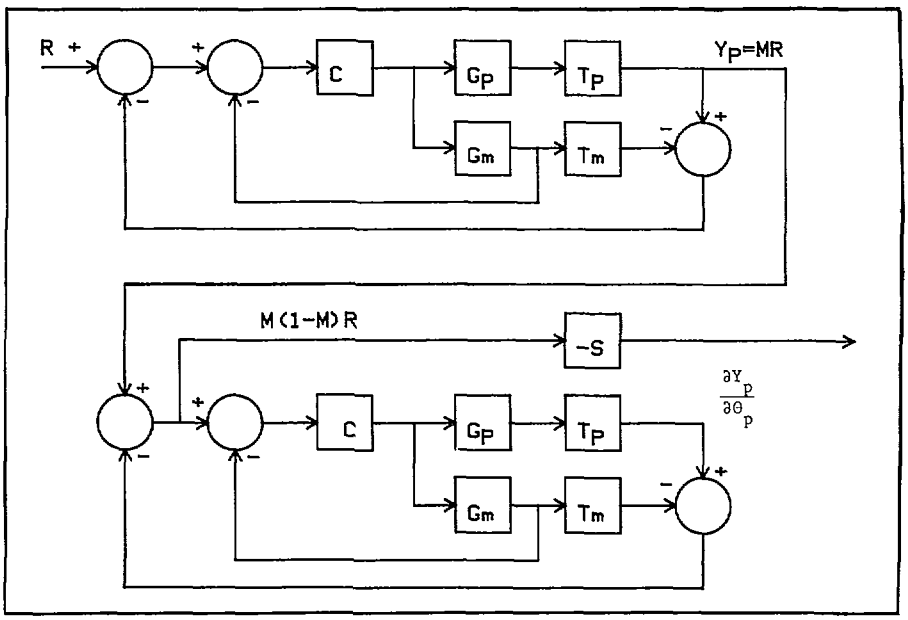


Fig. 7. Method of generating the sensitivity function $\partial y_p/\partial\theta_p$. Based on [3].

then both sides can be divided by $k\,\Delta\theta_p$ to yield

$$1/k = -\int_{t_1}^{t_2} \left|\frac{\partial y_p}{\partial \theta_p}\right|^2 dt \qquad (16)$$

If the $\partial y_p/\partial\theta_p$ is known, the usual case, then this constant could be precomputed. Otherwise, it would have to be calculated on line. This now gives the complete algorithm for changing the model time delay.

$$\Delta\theta_m = -k \int_{t_1}^{t_2} e \frac{\partial y_p}{\partial \theta_p} dt \qquad (17)$$

Fig. 8 shows this implementation.

Marshall [3] has simulated the system of Fig. 8 (which encompasses the system of Fig. 7) on both a hybrid computer and on a digital computer. It was stable and had satisfactory dynamics when the initial model time delay was wrong by as much as 80%. The system adjusted its model time delay to within 10% of the correct value in one time delay period, and settled at the correct value in two periods.

For most linear systems there is a maximum value of gain that will ensure stability. If a time delay is introduced into such a system, then the gain must be reduced. Use of a Smith predictor controller will allow the gain to be restored to its original value. However, the system still tracks targets with a time delay. Recently many embellishments for time-delay systems have been reported. There are ways of dealing with nonlinear time-delay systems [15]; there are ways of applying optimal control techniques to the Smith predictor scheme [3], [16]; and there are ways of making certain systems track targets with zero-latency [7], [8]. I will discuss only the last of these techniques.

Zero-latency Tracking In Time-delay Systems

In some tracking systems the target position can be predicted, and it is possible to produce zero-latency tracking. To do this one does not simply predict the future target position and feed this into the time-delay system. Instead, one computes an adaptation signal that depends on the target movement, as well as on the time delay and dynamics of the plant. This adaptation signal is applied to the time-delay system, and it allows zero-latency tracking [7], [8].

Figure 9 shows this scheme applied to a simple state-variable feedback control

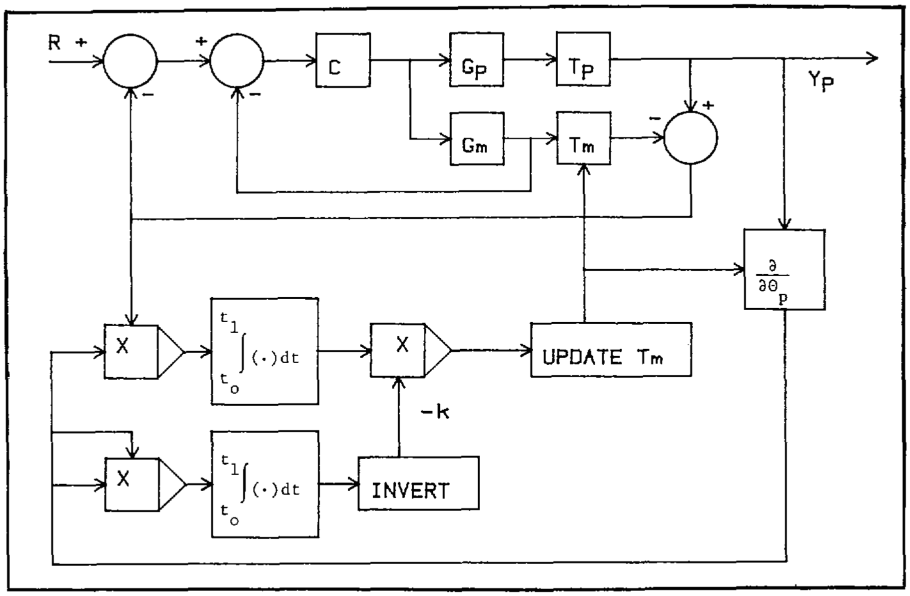


Fig. 8. An adaptive controller on a Smith predictor. Based on [3].

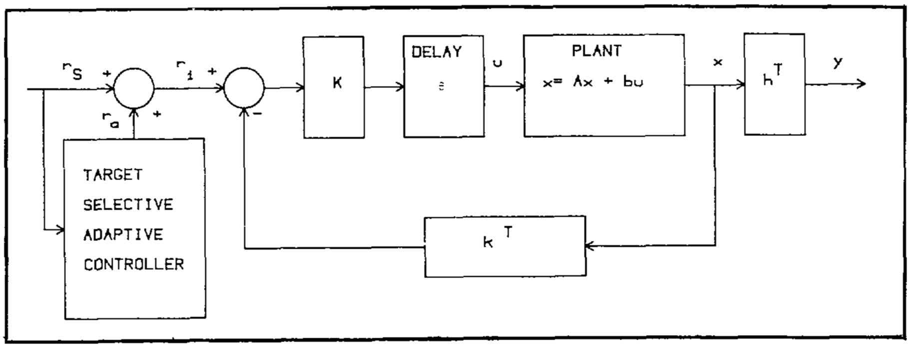


Fig. 9. The target-selective adaptive control scheme.

system with a time delay in the forward path. The system input, $r_i(t)$, is composed of two parts; the reference source, $r_s(t)$, and the adaptive signal, $r_a(t)$. When $r_s(t)$ is not a known target waveform, $r_a(t)$ is turned off; $r_i(t)$ then equals $r_s(t)$ and the closed-loop transfer function becomes

$$\frac{Y(s)}{R_i(s)} = \frac{h^T(sI-A)^{-1}bKe^{-s\theta}}{1+k^T(sI-A)^{-1}bKe^{-s\theta}} \qquad (18)$$

The $e^{-s\theta}$ term in the numerator is a pure time delay that remains in spite of the feedback. The $e^{-s\theta}$ term in the denominator produces the phase lag that reduces the allowable gain. The other symbols represent Y(s) (the scalar output), $R_i(s)$ (the scalar system input), $\mathbf{I}$ (the $n \times n$ identity matrix), $\mathbf{A}$ (the $n \times n$ system matrix), K (the scalar gain), $\mathbf{k}^T$ (the $1 \times n$ feedback control vector), $\mathbf{h}^T$ (the $1 \times n$ output coefficient vector), and $\mathbf{b}$ (the $n \times 1$ input coefficient vector). Superscript T indicates the vector transpose operation. The dimensions of the vectors and matrices are such that the numerator and the denominator of (18) are scalars. The feedback vector $\mathbf{k}^T$ and the gain K must be selected to achieve stability.

For zero-latency tracking the output must be identically equal to the reference input: $y(t) = r_s(t)$. Applying the requirement $Y(s) = R_s(s)$ to equation (18) produces:

$$R_s = \left[\frac{\mathbf{h}^T(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}Ke^{-s\theta}}{1+\mathbf{k}^T(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}Ke^{-s\theta}}\right] \cdot (R_s + R_a) \qquad (19)$$

Solving for R_a yields:

$$R_a = \left[\frac{e^{s\theta}}{\mathbf{h}^T(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}K} + \frac{\mathbf{k}^T(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}}{\mathbf{h}^T(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}} - 1\right] R_s \qquad (20)$$

If the time delay θ, the gain K, the matrix $\mathbf{A}$, and the vectors $\mathbf{b}$, $\mathbf{k}^T$, and $\mathbf{h}^T$ are known, and if $r_s(t)$ can be estimated, then $r_a(t)$ can be computed. The output can be made equal to the input, compensating for both the time delay and the plant dynamics. This control scheme has only been studied for single-input single-output systems with scalar K and only one time delay.

One reason for studying such zero-latency tracking is that the human eye movement control system seems to use target-selective adaptive control. The human oculomotor system has a 150 msec time delay. When a target starts moving there is a 150 msec delay before the eye starts moving. When the target stops, the eye continues to follow the predicted target for about 150 msec. However, a human can track a predictable target, such as the one shown in Fig. 10, without latency or phase lag. The top trace shows the *target* position (*dotted*) and the eye position (solid), and the bottom trace shows *target* velocity (*solid*) and eye velocity (dotted). The target moved ±5 degrees. The time axis is labeled in seconds. The target position waveform was that of a repeated cubic segment. A model was built to help explain how the human could overcome such a time delay and track with no latency. The model performed as well as the human [7], [8]. The model required knowledge about plant dynamics, plant time delay, target amplitude, target frequency and target waveform.

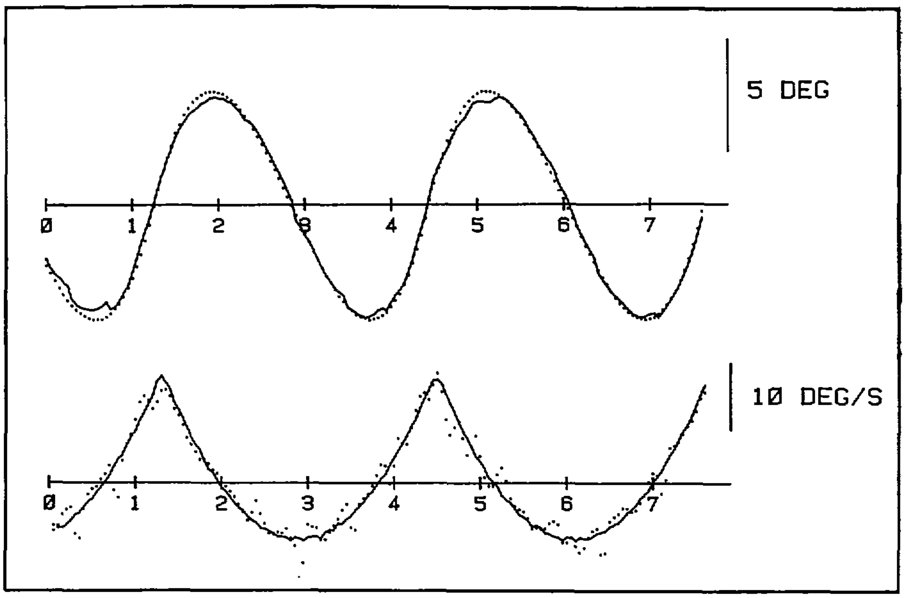


Fig. 10. Human zero-latency tracking of a cubic target waveform.

Conclusion

To maintain stability of a control system after a time delay is introduced, the gain must be reduced. The Smith predictor algorithm allows larger gains. However, it requires an exact matching of model and plant parameters. An adaptive control loop added to the Smith predictor can automatically adjust model parameters to match the time varying plant parameters. The resulting system still tracks targets with a time delay. In certain circumstances controllers can be designed to track targets with no latency in spite of plant time delays.

Acknowledgements

I thank Jeffery S. Kallman and two perceptive reviewers for constructive criticisms. This research was partially supported by National Science Foundation grant ECS-8121259.

References

[1] P. B. Deshpande and R. H. Ash, *Elements of Computer Process Control With Advanced Control Applications*. Research Triangle Park NC: Instrument Society of America, 1981, pp. 227–250.

[2] O. J. M. Smith, "Closer control of loops with dead time," *Chemical Engineering Progress*, vol. 53(5), pp. 217–219, 1957.

[3] J. E. Marshall, *Control of Time-Delay Systems*. Stevenage United Kingdom: Peter Peregrinus Ltd., 1979.

[4] A. C. Ioannides, G. J. Rogers and V. Latham, "Stability limits of a Smith controller in simple systems containing a time delay," *Int J Control*, vol. 29, pp. 557–563, 1979.

[5] Z. Palmor, "Stability properties of Smith dead-time compensator controllers," *Int J Control*, vol. 32, pp. 937–949, 1980.

[6] I. D. Landau, *Adaptive Control: The Model Reference Approach*. New York: Marcel Dekker Inc, 1979.

[7] A. T. Bahill and J. D. McDonald, "The smooth pursuit eye movement system uses an adaptive controller to track predictable targets," in *Proc Int Conf Cybern Soc*, New York: IEEE, 1981, pp. 269–278.

[8] J. D. McDonald, A. T. Bahill and M. B. Friedman, "An adaptive control model for human head and eye movements while walking," *IEEE Trans Syst Man Cybern*, vol. SMC-13, in press, 1983.

[9] J. M. Mendel, J. Kormylo, F. Aminzadeh, J. S. Lee and F. Habibi-Ashrafi, "A novel approach to seismic signal processing and modeling," *Geophysics*, vol. 46, pp. 1398–1414, 1981.

[10] G. J. Agin, "Real time control of a robot with a mobile camera," in *9th International Symposium on Industrial Robots*, March 1979: Washington D.C., 1979, pp. 57–70.

[11] F. L. N-Nagy and M. N. Al-Tikriti, "Stability criterion of linear control systems with time delay," *Meas Control*, vol. 3, pp. 86–87, 1970.

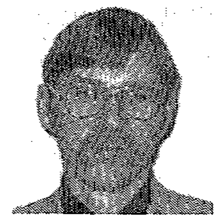
[12] K. S. Narendra and R. V. Monopoli, *Applications of Adaptive Control*. New York: Academic Press, 1980.

[13] J. W. Gilbart and G. C. Winston, "Adaptive compensations for an optical tracking telescope," *Automatica*, vol. 10, pp. 125–131, 1974.

[14] C. G. Kallstrom, K. J. Åstrom, N. E. Thorell, J. Eriksson and L. Sten, "Adaptive autopilots for tankers," *Automatica*, vol. 15, pp. 241–254, 1979.

[15] M. Jamshidi, "A three-stage design of non-linear control systems with time-delay," *Int J Control*, vol. 21, pp. 753–762, 1975.

[16] K. Watanabe and M. Ito, "A process-model control for linear systems with delay," *IEEE Trans Autom Control*, vol. AC-26, pp. 1261–1269, 1981.



A. Terry Bahill (S'66-M'68-SM'81) was born in Washington, Pennsylvania, on January 31, 1946. He received the B.S.E.E. degree in electrical engineering from the University of Arizona, Tucson, in 1967; the M.S.E.E. degree in electrical engineering from San Jose State Univesity, San Jose, California, in 1970; and the Ph.D. in electrical engineering and computer science from the University of California, Berkeley, in 1975. He served as a Lieutenant in the U.S. Navy teaching mathematics and electrical engineering for four years to the students of the Navy Nuclear Power School, Mare Island, California. He is an Associate Professor of Electrical and Biomedical Engineering at Carnegie-Mellon University and also an Adjunct Associate Professor of Neurology at the University of Pittsburgh Medical School. His research interests include control theory, modeling physiological systems, head and eye coordination, and computer text and data processing. He is the author of *Bioengineering: Biomedical, Medical, and Clinical Engineering* (Prentice-Hall, 1981).