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Describing baseball pitch movement with right-hand rules

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Abstract

The right-hand rules show the direction of the spin-induced deflection of baseball pitches: thus, they explain the movement of the fastball, curveball, slider and screwball. The direction of deflection is described by a pair of right-hand rules commonly used in science and engineering. Our new model for the magnitude of the lateral spin-induced deflection of the ball considers the orientation of the axis of rotation of the ball relative to the direction in which the ball is moving. This paper also describes how models based on somatic metaphors might provide variability in a pitcher's repertoire.

Keywords: Curveball; Pitch deflection; Screwball; Slider; Modeling; Forces on a baseball; Science of baseball

1. Introduction

If a major league baseball pitcher is asked to describe the flight of one of his pitches; he usually illustrates the trajectory using his pitching hand, much like a kid or a jet pilot demonstrating the yaw, pitch and roll of an airplane. The hand used as an analog in this way is a gestural example of a *somatic metaphor* [1]. Edelman [2] writes, "The shape and feel of the body as it moves and interacts with the environment play key roles in the building up of a sense of space and of the possibilities of action." Like other kinds of analogies, the somatic metaphor helps a modeler form a conceptual system to deal with the external world [3,4].

The right-hand rules form a pair of gestural metaphors that has been widely used for centuries as mnemonic or heuristic devices in science, mathematics and engineering. Unlike the somatic metaphor used by the baseball pitcher to describe the trajectory of his pitch, these rules have been formalized to increase accuracy and repeatability. This pair comprises an *angular right-hand rule* and a *coordinate right-hand rule*. The angular rule describes angular relationships of entities relative to a given axis and the coordinate rule establishes a local coordinate system, often based on the axis derived from the angular rule.

Well-known examples of right-hand rules used in science and engineering include those specifying the force on a wire carrying current in a magnetic field, the direction of current for a wire moving in a magnetic field, the angular moment of a force about an axis, the direction of torque, the direction of curvatures of DNA and protein molecule helices, the direction of momentum of an antineutrino, the direction of spin of subatomic particles, the orientation of the cross product of vectors, and translations and rotations of screws and coils. Maxwell [5] suggests that the gestural metaphor "… will impress the right-handed screw motion on the memory more firmly than any verbal definition."

Right-hand rules can also be used in sports engineering to describe the direction of deflection of a baseball pitch. Because the local right-hand coordinate system specified by these rules is independent of any global coordinate systems, the right-hand rules apply to pitches of both right-handed and left-handed pitchers. In this paper, we develop and describe these rules with the hope that they might be useful to help pitchers understand what they are doing to make the ball curve.

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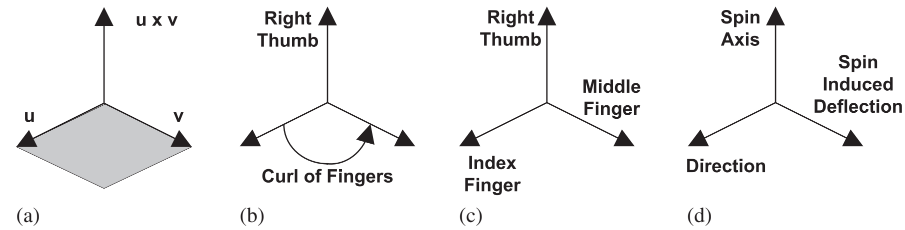

Fig. 1. (a) The vector (or cross) product of vectors **u** and **v** is perpendicular to the plane of **u** and **v**. (b) The angular right-hand rule. If the fingers of the right hand are curled in the direction from **u** to **v**, the thumb will point in the direction of the vector $\mathbf{u} \times \mathbf{v}$, which is pronounced **u** cross **v**. (c) The coordinate right-hand rule. The index finger, the middle finger and the thumb point in the directions of **u**, **v**, and $\mathbf{u} \times \mathbf{v}$, respectively. (d) For a baseball, the cross product of the spin axis and the direction of motion gives the direction of the spin-induced deflection. Copyright©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

2. Right-hand rules for the pitch

Batters say that the ball hops, drops, curves, breaks, rises, sails or tails away. The pitcher might tell you that he throws a fastball, screwball, curveball, drop curve, flat curve, slider, backup slider, change up, split fingered fastball, splitter, forkball, sinker, cutter, two-seam fastball or four-seam fastball. This sounds like a lot of variation. However, no matter how the pitcher grips or throws the ball, once it is in the air its motion depends only on gravity, its velocity and its spin.[1] In engineering notation, these pitch characteristics are described, respectively, by a *linear velocity vector* and an *angular velocity vector*, each with magnitude and direction. The magnitude of the linear velocity vector is called *pitch speed* and the magnitude of the angular velocity vector is called the *spin rate*. These vectors produce a force acting on the ball that causes a deflection of the ball's trajectory. This lateral force produced by the spin of a moving ball is called the Magnus force, because Gustav Magnus was one of the first scientists to study this effect [6]. Two models explain the origin of this Magnus force, one based on conservation of momentum and the other based on Bernoulli's principle [7,8]. We will apply the right-hand rules to the linear velocity vector and the angular velocity vector in order to describe the direction of the spin-induced deflection of the pitch.

2.1. Right-hand rules and the cross product

In vector analysis, the right-hand rules specify the orientation of the cross product of two vectors. Fig. 1 (a) shows that the cross (or vector) product, written as $\mathbf{u} \times \mathbf{v}$, of non-parallel vectors **u** and **v** is perpendicular to the plane of **u** and **v**: the symbol $\times$ represents the cross product. The angular right-hand rule, illustrated in Fig. 1(b), is used to specify the orientation of a cross product $\mathbf{u} \times \mathbf{v}$. If the fingers of the right-hand are curled in the direction from **u** to **v**, the thumb will point in the direction of the vector $\mathbf{u} \times \mathbf{v}$. The coordinate right-hand rule is illustrated in Fig. 1(c). The index finger, middle finger and thumb point in the directions of **u**, **v** and $\mathbf{u} \times \mathbf{v}$, respectively, in this local coordinate system. The vectors of Fig. 1(d) represent the angular velocity vector (spin), the linear velocity vector (direction) and the spin-induced deflection force of a spinning pitch.

2.2. Right-hand rules applied to the spinning ball

The spin axis of the pitch can be found by using the angular right-hand rule. As shown in Fig. 2, if you curl the fingers of your right hand in the direction of spin, your extended thumb will point in the direction of the spin axis.

The direction of the spin-induced deflection force can be described using the coordinate right-hand rule. Point the thumb of your right hand in the direction of the spin axis (as determined from the angular right-hand rule), and point your index finger in the direction of forward motion of the pitch (Fig. 3). Bend your middle finger so that it is perpendicular to your index finger. Your middle finger will be pointing in the direction of the spin-induced deflection (of course, the ball also drops due to gravity). The spin-induced deflection force will be in a direction represented by the cross product of the angular velocity vector and the linear velocity vector of the ball: angular velocity $\times$ linear velocity $=$ spin-induced deflection force. Or mnemonically, spin axis $\times$ direction $=$ spin-induced deflection (SaD Sid). This acronym only gives the direction of deflection. The equation yielding the magnitude of the spin-induced deflection force is more complicated and is discussed in Section 2.4.

2.3. Deflection of specific kinds of pitches

Figs. 4 and 5 show the directions of spin (circular arrows) and spin axes[2] (straight arrows) of some common pitches from the perspective of the pitcher (Fig. 4 represents a right-hander's view and Fig. 5 a left-hander's view). We will now consider the direction of deflection of each of these pitches.

Fig. 4 illustrates the fastball, curveball and slider, distinguished by the direction of the spin axis. When a layperson

[1] This statement is true even for the knuckleball, because it is the shifting position of the seams and the spin axis during its slow spin in route to the plate that gives the ball its erratic behavior. Air density and ball size produce small second order effects.

[2] These could be labeled spin vectors, because they suggest both magnitude and direction.

Fig. 2. The angular right-hand rule. For a rotating object, if the fingers are curled in the direction of rotation, the thumb points in the direction of the spin axis. Photo by Zach Bahill. Copyright©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

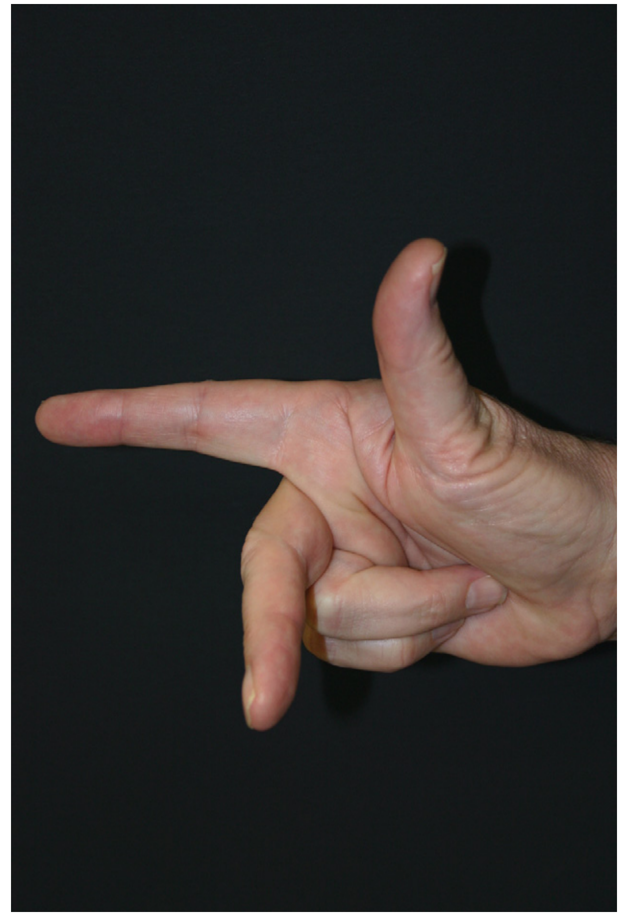

Fig. 3. The coordinate right-hand rule. For a baseball, if the thumb points in the direction of the spin axis and the index finger points in the direction of forward motion of the pitch, then the middle finger will point in the direction of the spin-induced deflection. Photo by Zach Bahill. Copyright©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

throws a ball, the fingers are the last part of the hand to touch the ball. They touch the ball on the bottom and thus impart backspin to the ball. Most pitchers throw the fastball with a three-quarter arm delivery, which means the arm does not come straight over-the-top, but rather it is in between over the top and sidearm. This delivery rotates the spin axis from the horizontal as shown in Fig. 4. The curveball is also thrown with a three-quarter arm delivery, but this time the pitcher rolls his wrist and causes the fingers to sweep in front of the ball. This produces a spin axis as shown for the curveball of Fig. 4. This pitch will curve at an angle from upper right to lower left as seen by a right-handed pitcher. Thus, the ball curves diagonally. The advantage of the drop in a pitch is that the sweet area of the bat is about four inches long (10 cm) [9] but only one-third of an inch (8 mm) high [10]. Thus, a vertical drop is more effective than a horizontal curve at taking the ball away from the bat's sweet area.

The overarm fastball shown in Fig. 5 has a predominate backspin, which gives it lift, thereby decreasing its fall due to gravity. But when the fastball is thrown with a three-quarter

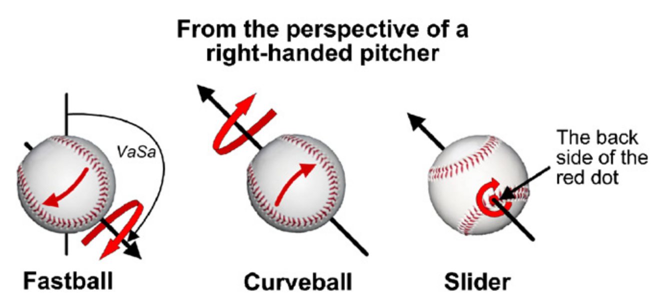

Fig. 4. The direction of spin (circular arrows) and the spin axes (straight arrows) of a *three-quarter arm* fastball, a curveball and a slider from the perspective of a *right*-handed pitcher, meaning the ball is moving into the page. VaSa is the angle between the *V*ertical *a*xis and the *S*pin *a*xis (VaSa). Copyright©, 2005, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

arm delivery (as in Fig. 4), the lift is reduced but it introduces lateral deflection (to the right for a right-handed pitcher). A sidearm fastball (from a lefty or a righty) tends to have some topspin, because the fingers put pressure on the top half of the ball during the pitcher's release. The resulting deflection augments the effects of gravity and the pitch "sinks."

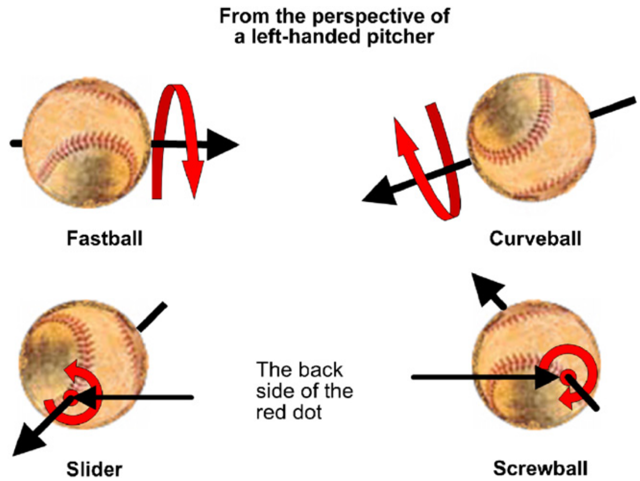

Fig. 5. The direction of spin (circular arrows) and the spin axes (straight arrows) of an *overarm* fastball, a curveball, a slider and a screwball from the perspective of a *left*-handed pitcher, meaning the ball is moving into the page. Copyright©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

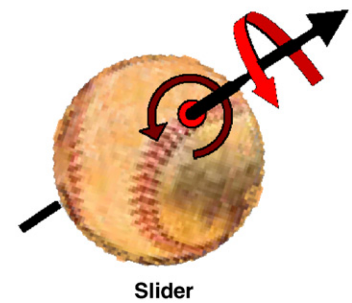

Fig. 6. The batter's view of a slider thrown by a right-handed pitcher: the ball is coming out of the page. The red dot signals the batter that the pitch is a slider. Copyright©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

The slider is thrown somewhat like a football. Unlike the fastball and curveball, the spin axis of the slider is not perpendicular to the direction of forward motion (although the direction of deflection is still perpendicular to the cross product of the spin axis and the direction of motion). As the angle between the spin axis and the direction of motion decreases, the magnitude of deflection decreases, but the direction of deflection remains the same. If the spin axis is coincident with the direction of motion, as for the backup slider, the ball spins like a bullet and undergoes no deflection.[3] Therefore, the slider is usually thrown so that the axis of rotation is pointed up and to the left (from the perspective of a right-handed pitcher). This causes the ball to drop and curve from the right to the left. Rotation about this axis allows some batters to see a red dot at the spin axis on the top-right-side of the ball (see Fig. 6). Bahill et al. [11] show pictures of this spinning red dot. Seeing this red dot is important, because if the batter can see this red dot, then he will know that the pitch is a slider and he can therefore better predict its trajectory. We questioned 15 former major league hitters; eight remembered seeing this dot, but two said it was black or dark gray rather than red. For the backup slider, the spin causes no horizontal deflection and the batter might see a red dot in the middle of the ball.

2.4. *Forces acting on the ball in flight*

This section has equations, but it can be skipped without loss of continuity. A baseball in flight is influenced by three forces: gravity pulling downward, air resistance or drag operating in the opposite direction of the ball's motion and, if it is spinning, a force perpendicular to the direction of motion. The force of gravity is downward, where m is the mass of the ball and g is the gravitation constant: its magnitude is the ball's weight, 0.32 lb. The magnitude of the force opposite to the direction of flight is

$$F_{\mathrm{drag}} = 0.5\rho\pi r_{\mathrm{ball}}^2 v_{\mathrm{ball}}^2 C_{\mathrm{d}}, \tag{1}$$

where ρ is air mass density, v_{ball} is the ball speed and r_{ball} is the radius of the ball [7, p. 168]. Typical values for these parameters are given in Table 1. Of course SI units can be used in this equation, but if English units are to be used in Eqs. (1)–(7) then ρ is in lb s^2/ft^4, v_{ball} is in ft/s, r_{ball} is in ft, F_{drag} is in ft and in later equations ω is in rad/s. For the drag coefficient, C_{d}, we use a value of 0.5. This drag coefficient is discussed in the 2.8 *Modeling philosophy* section of this paper.

The earliest experimental equation for the transverse force on a spinning object moving in a fluid is the Kutta–Joukowski Lift Theorem

$$\mathbf{L} = \rho \mathbf{U} \times \boldsymbol{\Gamma}, \tag{2}$$

where $\mathbf{L}$ is the lift force per unit length of cylinder, ρ is the fluid density, $\mathbf{U}$ is the fluid velocity and $\boldsymbol{\Gamma}$ is the circulation around the cylinder. $\mathbf{L}$, $\mathbf{U}$ and $\boldsymbol{\Gamma}$ are vectors. When this equation is tailored for a baseball [7, pp. 77–81], we get the magnitude of the spin-induced force acting perpendicular to the direction of flight

$$F_{\mathrm{perpendicular}} = 0.5\rho\pi\omega r_{\mathrm{ball}}^3 v_{\mathrm{ball}}, \tag{3}$$

where ω is the spin rate. This is often called the Magnus force. This force can be decomposed into a force lifting the ball up and a lateral force pushing it sideways

$$F_{\mathrm{upward}} = 0.5\rho\pi\omega r_{\mathrm{ball}}^3 v_{\mathrm{ball}} \sin \mathrm{VaSa}, \tag{4}$$

where VaSa is the angle between the vertical axis and the spin axis (Figs. 4 and 7). The magnitude of the lateral force is

$$F_{\mathrm{sideways}} = 0.5\rho\pi\omega r_{\mathrm{ball}}^3 v_{\mathrm{ball}} \cos \mathrm{VaSa}. \tag{5}$$

Finally, if the spin axis is not perpendicular to the direction of motion (as in the case of the slider), the magnitude of the cross

[3] We believe the following explains the origin of the name *backup slider*. The pitcher and catcher agree on a slider. The catcher expects the ball to drop and move to his right, for a right-handed pitcher. Therefore, during the pitch, the catcher's glove will be moving down and to the right. But if the pitcher fails to point the spin axis up, then the ball will not deflect down and to the right and the catcher has to *backup* his glove in order to catch the ball.

Table 1
Typical baseball and softball parameters for line drives

	Major league baseball	Little league	NCAA softball
Ball	Hardball	Hardball	Softball
Ball weight (oz)	5.125	5.125	6.75
Ball weight, F (gravity), (lb)	0.32	0.32	0.32
Ball radius (in)	1.45	1.45	1.9
Ball radius, r_{ball} (ft)	0.12	0.12	0.16
Pitch speed (mph)	85	50	65
Pitch speed, v_{ball} (ft/s)	125	73	95
Distance from front of rubber to tip of plate (ft)	60.5	46	43
Pitcher's release point: (distance from tip of plate, height), (ft)	(55.5, 6)	(42.5, 5)	(40.5, 2.5)
Bat–ball collision point: (distance from tip of plate, height), (ft)	(3, 3)	(3, 3)	(3, 3)
Air weight density, (lb/ft^3)	0.074	0.074	0.074
Air mass density, ρ (lb s^2/ft^4)	0.0023	0.0023	0.0023
Bat	Wooden C243	Aluminum	Aluminum
Bat weight (oz)	32	23	25
Maximum bat radius (in)	1.25	1.125	1.125
Speed of sweet spot (mph)	60	45	50
Coefficient of restitution	0.54	0.60	0.52
Backspin of batted ball (rps)	10–70	10–70	10–70
Desired ground contact point from the plate (ft)	90–200	70–150	70–160

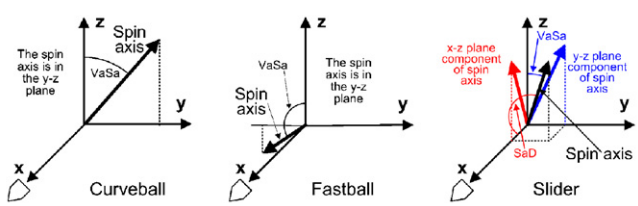

Fig. 7. Rectangular coordinate system and illustration of the angles VaSa and SaD for a curveball, a three-quarter arm fastball and a slider all thrown by a right-handed pitcher. The origin is the pitcher's release point. For the curveball, the spin axis is in the *y–z* plane. For the fastball the spin axis is also in the *y–z* plane, but is it is below the *y*-axis. For the slider, the spin axis has components in both the *y–z* and *x–z* planes. Copyright©, 2006, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

product of these two vectors will depend on the angle between the spin axis and direction of motion, this angle is called SaD. In aeronautics it is called the angle of attack.

$$F_{\text{lift}} = 0.5\rho\pi\omega r_{\text{ball}}^3 v_{\text{ball}} \sin \text{VaSa} \sin \text{SaD}, \tag{6}$$

$$F_{\text{lateral}} = 0.5\rho\pi\omega r_{\text{ball}}^3 v_{\text{ball}} \cos \text{VaSa} \sin \text{SaD}. \tag{7}$$

During the pitch, gravity is continuously pulling the ball downward, which changes the direction of motion of the ball by 5–10° during its flight. However, the ball acts like a gyroscope, so the spin axis does not change. This means that, for a slider, the angle SaD increases and the forces of Eqs. (6) and (7) may increase by as much as 10% during the ball's flight.

2.5. Comparison of the slider and curveball

Let us now compare the magnitude of this lateral spin-induced deflection force (Eq. (7)) for two specific pitches, namely the slider and the curveball. The magnitude of the lateral spin-induced deflection of the slider is less than that of a curveball for the following four reasons.

(1) For the curveball, the angle between the spin axis and the direction of motion (SaD) is around 85°. For the slider, it is around 60°. The magnitude of the cross product is proportional to the sine of this angle. Therefore, the slider's deflection force is less than the curveball's by the ratio $\sin 60/\sin 85$: the slider force equals 0.87 times the curveball force. The angle between the vertical axis and the spin axis (VaSa) has no effect because it is about the same for the slider and the curveball.

(2) The curveball spins at up to 33 revolutions per second (rps) and the slider probably spins around 23 rps [12], so the slider's deflection force is smaller because of its slower rotation: the slider force equals 0.7 times the curveball force.

(3) The deflection force also depends on the speed of the pitch. Assume a 75 mph (33.5 m/s) curveball and an 85 mph (38.0 m/s) slider: the slider force equals 1.13 times the curveball force.

Therefore, for the three effects of this example, the total slider force equals 0.69 times the curveball force.

(4) Furthermore, the curveball is slower, so it is in the air longer. Therefore, the deflection force has to operate longer and the total deflection due to this effect is greater. An 85 mph slider travels from the pitcher's release point (5 ft in front of the rubber) to the point of bat–ball collision (1.5 ft in front of the plate) in 453 ms, whereas a 75 mph curveball is in the air 513 ms. The total deflection is proportional to total force times duration squared: therefore, the ratio deflection of the slider with respect to the curveball is, ratio $\text{Force}_{\text{spin-axis}} * \text{ratio Force}_{\text{spin-rate}} * \text{ratio Force}_{\text{speed}} * \text{ratio durations squared} = 0.87 * 0.7 * 1.13 * 0.78 = 0.54$. In summary, the magnitude of the lateral spin-induced deflection of the slider is about half that of the curveball.

Table 2
Gravity-induced and spin-induced drop (a) (with English units) and (b) (with SI units)

Pitch type and speed	Spin rate (rps)	Duration of flight (ms)	Drop due to gravity (ft)	Spin-induced vertical drop (ft)	Total drop (ft)
(a)					
95 mph fastball	−20	404	2.63	−0.91	1.72
90 mph fastball	−20	426	2.92	−0.98	1.94
85 mph slider	+23	452	3.29	+0.74	4.03
80 mph curveball	+33	480	3.71	+1.40	5.11
75 mph curveball	+33	513	4.24	+1.46	5.70
Pitch type and speed	Spin rate, ω (rad/s)	Duration of flight (ms)	Drop due to gravity (m)	Spin-induced vertical drop (m)	Total drop (m)
(b)					
42.5 m/s fastball	−126	404	0.80	−0.28	0.52
40.2 m/s fastball	−126	426	0.89	−0.30	0.59
38.0 m/s slider	+145	452	0.95	+0.23	1.23
35.8 m/s curveball	+207	480	1.13	+0.43	1.56
33.5 m/s curveball	+207	513	1.29	+0.45	1.74

The screwball (sometimes called a “fadeaway” or “in-shoot”) was made popular in the early 1900s by Christy Mathewson and Mordecai “Three Fingered” Brown[4] and was repopularized by the left-hander Carl Hubbell in the 1930s. Therefore, we show it from the left-hander’s perspective in Fig. 5. Of the pitches shown in Figs. 4 and 5, it is the least-used, in part, because the required extended pronation of the hand strains the forearm and elbow. At release, the fingers are on the inside and top of the ball. The deflection of the left-hander’s screwball is the same as the deflection of a right-hander’s slider. The screwball is basically a slider, so its deflection will be less than that of a curveball for the reasons given above.

The direction of deflection of these pitches is variable depending on the direction of the spin axis. The direction of this axis varies with the angle of the arm during delivery and the position of the fingers on the ball at the time of release. This is how the pitcher controls the direction of deflection.

2.6. Vertical deflection

Table 2 shows the magnitude of the spin-induced drop for three kinds of pitches at various speeds, as determined by simulation. Our baseball trajectory simulator includes the effects of lift and drag due to spin on the ball [8,13]. A 90 mph (40.2 m/s) fastball is in the air 426 ms, so it drops 2.92 ft (0.89 m) due to gravity ($1/2gt^2$, where the gravitational constant $g = 32.2\,\text{ft/s}^2$ ($9.8\,\text{m/s}^2$) and t = time from release until the point of bat–ball collision). But the backspin lifts this pitch 0.98 ft (0.3 m), producing a total drop of 1.94 ft (0.59 m) as shown in Table 2. Negative numbers in spin-induced vertical drop column of Table 2 mean the ball is being lifted up. All of the pitches in Table 2 were launched horizontally—that is, with a launch angle of zero. The angle VaSa was also set to zero: therefore, pitches thrown with a three-quarter arm delivery would have smaller spin-induced deflections than given in Table 2.

[4] Due to childhood accidents, Brown had a deformed pitching hand that gave him a “natural” screwball.

Vertical misjudgment of the potential bat–ball impact point is a common cause of batters’ failure to hit safely [10,14]. The vertical differences between the curveballs and fastballs in Table 2 are greater than 3 ft (0.9 m), whereas the difference between the two speeds of fastball is about 3 in (7.6 cm) and the difference between the two speeds of curveball is a little greater than 7 in (17.8 cm). However, the batter is more likely to make a vertical error because speed has been misjudged than because the kind of pitch has been misjudged [10,14]. A vertical error of as little as one-third of an inch (0.8 cm) in the batter’s swing will generally result in an out [10].

The spin on the pitch also causes a horizontal deflection of the ball. In *deciding* whether to swing, the horizontal deflection is more important than the vertical, because the umpire’s judgment with respect to the corners of the plate has more precision than his or her judgment regarding the top and bottom of the strike zone. However, after the batter has decided to swing and is trying to *track and hit* the ball, the vertical deflection becomes more important.

The right-hand rules for the lateral deflection of a spinning ball also apply to the batted ball, except it is harder to make predictions about the magnitude of deflection because we have no data about the spin on a batted ball. The right-hand rules can be applied to tennis, where deflections are similar to baseball, but not to football, because spin-induced deflections of a football are small [15].

2.7. Modeling philosophy

Although our equations and discussion might imply great confidence and precision in our numbers, it is important to note that our equations are only models. The Kutta–Joukowski equation and subsequent derivations are not theoretical equations, they are only approximations fit to experimental data. There are more complicated equations for the forces on a baseball (e.g. see [16–19]). Furthermore, there is much that we did not include in our model. We ignored the possibility that air flowing around certain areas of the ball might change from turbulent to laminar flow in route to the plate. Our equations did not in-

Table 3
Typical values for major league pitches

Type of pitch	Initial speed (mph)	Spin rate (rpm)	Spin rate (revolution/s)	Rotations between pitcher's release and the point of bat–ball contact
Fastball	85–95	1200	20	8
Slider	80–85	1400	23	10
Curveball	70–80	2000	33	17
Palmball (a changeup)	60–70	400	7	4
Knuckle ball	60–70	30	1/2	1/4

clude effects of shifting the wake of turbulent air behind the ball. The ball loses about 10% of its linear velocity in route to the plate: so it probably also loses angular velocity; we did not model this. We ignored the difference between the center of gravity and the geometrical center of the baseball [20]. We ignored possible differences in the moments of inertia of different balls. We ignored the precession of the spin axis. In computing velocities due to bat–ball collisions, we ignored deformation of the ball and energy dissipated when the ball slips across the bat surface. Finally, as we have already stated, we treated the drag coefficient as a constant.

We used a value of 0.5 for the drag coefficient, C_d. However, for speeds over 80 mph this drag coefficient may be smaller [21, 7, p. 157,16,19]. There are no wind-tunnel data showing the drag coefficient of a spinning baseball over the range of velocities and spin rates that characterize a major league pitch. Sawicki et al. [18] summarize data from a half-dozen studies of spinning baseballs, non-spinning baseballs and other balls and showed C_d between 0.15 and 0.5. In most of these studies, the value of C_d depended on the speed of the airflow. In the data of [22], the drag coefficient can be fit with a straight line of $C_d = 0.45$, although there is considerable scatter in the data. The drag force causes the ball to lose about 10% of its speed in route to the plate. The simulations of [23] also studied this loss in speed. Data shown in their Fig. 8 for the speed lost in route to the plate can be nicely fitted with $\text{PercentSpeedLost} = 20C_d$, which implies $C_d = 0.5$.

It is somewhat surprising that given the multitude of modern computer–camera pitch tracking devices such as the QuesTec system, the best published experimental data for the spin rate of different pitched baseballs comes from Selin's cinematic measurements of baseball pitches [12]. And we have no experimental data for the spin on the batted-ball. Table 3 summarizes our best estimates of speed and spin rates for most popular major league pitches.

There is uncertainty in the numerical values used for the parameters in our equations. However, the predictions of the equations match baseball trajectories quite well. When better experimental data become available for parameters such as C_d and spin rate, then values of other parameters will have to be adjusted to maintain the match between the equations and actual baseball trajectories.

The value of this present study lies in comparisons rather than absolute numbers. (We have presented numerical values for rates and distances for sake of illustration only.) Our model emphasizes that the right-hand rules show the direction of the spin-induced deflections of a pitch. The model provides predictive power and comparative evaluations relative to the behavior of all kinds of pitches.

Stark [24] explained that models are ephemeral: they are created, they explain a phenomenon, they stimulate discussion, they foment alternatives and then they are replaced by new models. When there are better wind-tunnel data for the forces on a spinning baseball, then our equations for the lift and drag forces on a baseball will be supplanted by newer parameters and equations. But we think our models, based on the right-hand rules showing the direction of the spin-induced deflections, will have permanence: they are not likely to be superseded.

3. Somatic metaphors of pitchers

A pitcher uses his hand as a metaphor for the ball when asked to demonstrate the trajectory of a particular kind of pitch (such as a screwball). But he derives a mental model of a specific pitch from the feelings of arm angle and his fingers on the ball as the pitch is being released. By imagining slight shifts in these sensations, the pitcher can create subtly differing models that can provide pitch variability in his repertoire. For example, he might model the screwball with fingers on top of the ball when it is released (resulting in a downward deflection) or with fingers on the side of the ball (resulting in a flatter deflection).

The benefit of using the right-hand rules is that they simplify a dynamic process by providing static metaphors. These simplified models can help the pitcher understand the physics of pitching and should allow him or her to readily see how direction and magnitude of spin-induced deflection can be controlled by the pitcher's arm angle and pitch release characteristics.

For example, the somatic metaphor can help a pitcher understand how the slider will behave as a result of a given imparted rotation. The pitcher should note that when applying the right-hand rule to this pitch, the angle between the thumb (spin axis) and index finger (forward direction of pitch) is less than 90° for the slider, whereas it is near 90° for both the fastball and the curveball. The pitcher should realize that the middle finger (direction of deflection) continues to be perpendicular to both the thumb and index finger, but the magnitude of the deflection will decrease to zero as the angle between thumb and index finger decreases to zero. Thus, the right-hand rules for pitch aerodynamics account for the tilted spin axis of the slider. As the slider increasingly tilts forward, deflection of the pitch is diminished, resulting in a backup slider at the extreme. (As noted previously, both right-handed and left-handed pitchers should use their right hands to show the movement of the ball.)

It may be difficult for the batter to detect differences and therefore accurately predict different spin-induced deflections of different pitches. For example, a 95 mph (42.5 m/s) fastball thrown directly overarm looks much like a 95 mph fastball thrown with the arm angle lowered by 20°. However, the vertical difference in the potential bat–ball contact point is significant. For the 95 mph fastball with a 20-rps backspin shown in Table 2 (in addition to having a different release point), the pitch thrown with the lower arm angle would drop about three-quarters of an inch (1.9 cm) farther than the overarm pitch. Three-quarters of an inch is bigger than the vertical sweet spot. Mental models of pitch differences allow the pitcher to take advantage of the batter's difficulty in recognizing a wide variety of spin directions and detecting small shifts in arm angle. Therefore, the batter must detect the pitcher's release point, but he must also detect the ball's speed and spin in order to predict when and where the ball will cross the plate.

4. Summary

Somatic metaphors are pervasive in everyday life, so it is not surprising to find that baseball pitchers make use of these modeling devices in their work. We have shown how a pair of widely used engineering metaphors, the right-hand rules, provides a formalized approach to describing the pitchers' mental models, allowing prediction of the deflection direction of each pitch. Besides describing the behavior of the pitched ball, these rules can also be used to describe the deflection direction of the batted-ball. To determine the direction of deflection of the pitched-ball or the batted-ball, point the thumb of your right hand in the direction of the spin axis and your index finger in the direction of motion of the ball; your middle finger will indicate the direction of the spin-induced deflection (SaD Sid).

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