Journal of Experimental Psychology:
Human Perception and Performance
1993, Vol. 19, No. 1, 3–14

0096-1523/93/$3.00

The Perceptual Illusion of Baseball's Rising Fastball and Breaking Curveball

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The rising fastball and the breaking curveball are impossible according to principles of physics and physiology, yet many baseball players claim they exist. The simulation and model presented suggest that the rising fastball and breaking curveball are perceptual illusions caused by the batter misestimating the speed of the pitch. This model uses signals from known primary visual processes only. This model was enhanced by adding an acceleration term. The enhanced model more accurately predicts the position of the ball when it crosses the plate. These models are compared and contrasted to models by McBeath and Bootsma. Sensitivity analyses have shown that the model and simulation are robust with respect to their parameter values. The model is most sensitive to the estimated distance to the ball, and the simulation is most sensitive to the estimated speed of the pitch.

According to principles of physics and physiology, a rising fastball is impossible. Yet most batters claim it exists; they say the ball sometimes jumps a foot, right in front of home plate, causing the ball to hop over their bat. Our simulation and model explain this contradiction. Although the numbers given in this article are for professional baseball players, the simulation and model apply to all players, right down to Little Leaguers. They can also be extended to cricket.

The rising fastball could be defined as a pitch where the ball (a) jumps up, right in front of the plate, (b) crosses the plate above the pitcher's release point, (c) is going upward when it crosses the plate, or (d) falls less than would be expected due to gravity. By Definitions (b) and (c), a rising fastball could be thrown by a sidearm baseball pitcher or a softball pitcher, but not by an overhand baseball pitcher. For example, an overhand pitch is released about 6 ft (1.83 m) above the ground; if the ball crossed the plate higher than this it would not be a strike. And for the ball to be going upward when it crosses the plate, it would have to fall initially, and then near the end of its flight, experience an upward force that is greater than that of gravity. A force that opposes gravity is produced by the backspin on a fastball. However, the maximum spin rate ever measured for a human pitch, 2,300 revolutions per minute (rpm), would create a force only two thirds that of gravity (Watts & Bahill, 1990). So, although a fastball's lift from backspin may not overcome gravity, it does make it fall less than would be expected because of gravity, which is Definition (d). Therefore all fastballs fit this definition, making it trivially simple and uninteresting. Therefore, for the rest of this article we will only consider overhand pitches in baseball and Definition (a). For more discussion about batters' perceptions of the rising fastball, see McBeath (1990), who with 62 references has summarized the comments of baseball players and scientists over the last three centuries.

The Simulation

Figure 1a and the left half of Table 1 show our simulation of 95- and 90-mile-per-hour (mph) (43 and 40 m/s) simplified fastballs, based on Watts and Bahill (1990). (The right half of Table 1 will be explained later.) In this simulation, both pitches were launched horizontally (i.e., 0°), and the effects of air resistance and the forces resulting from spin were ignored. These simplifications are valid for the comparisons we are going to make. As shown in Figure 1d, the distance between the front of the pitcher's rubber and the tip of the plate is 60.5 ft (18.45 m). We assume that the pitcher releases the ball 5 ft (1.52 m) in front of the rubber. Therefore, for this simulation, the pitcher's release point was 55.5 ft (16.92 m) away from the tip of the plate. The bat hits the ball about 1.5 ft (0.46 m) in front of the batter's head, which was assumed to be aligned with the front of the plate. The plate measures 17 in. (43.18 cm) from the tip to the front edge. So in these simulations the point of bat–ball collision was 3 ft (0.91 m) in front of the tip of the plate, which is represented by the last entries of each column of Table 1. The pitcher's release point was assumed to be 6 ft (1.83 m) high. In this article, we will show how sensitive our conclusions are to these numbers.

The Qualitative Model for the Rising Fastball

As was first suggested by McBeath (1990), the illusion of the rising fastball could be the result of the batter underes-

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We thank Dale Milton, Manager of the Dodgers of the CDO Little League, for helping us measure our batters' performance. We also thank Reinoud Bootsma, Bill Ittelson, Michael McBeath, Dennis Proffitt, and David Regan for helpful comments about the article.

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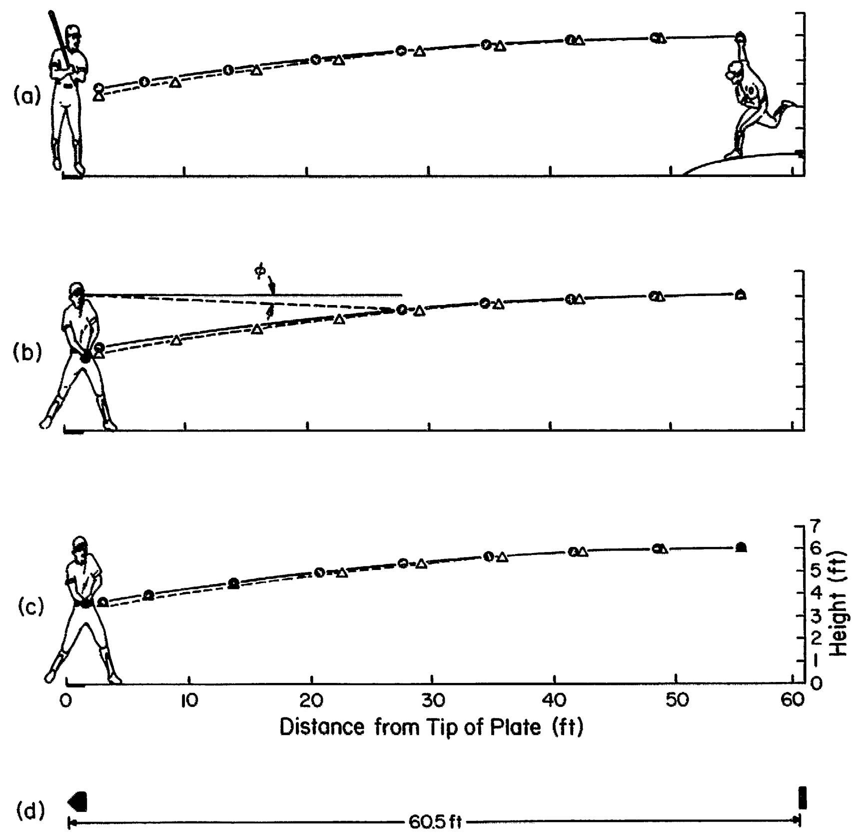

Figure 1. Panel a shows a computer simulation of the trajectory of a 95-mile-per-hour (mph; 43 m/s) fastball (solid line and circles) and a 90-mph (40 m/s) fastball (dashed line and triangles). (The slower pitch takes longer to get to the plate and therefore drops more.) Panel b shows a computer simulation of the trajectory of a 95-mph (43 m/s) fastball (solid line and circles), and the batter's mental model of this trajectory (dashed line and triangles) when he underestimated the speed of the pitch by 5 mph (2.2 m/s). (The batter's line of sight for one of these points is shown with a dashed line. The angle labeled φ indicates the angle of the batter's line of sight.) Panel c shows the same simulation as Figure 1b, except that when the ball was 20 ft (6.1 m) in front of the plate, the "batter" realized his mental model was wrong and corrected it, thus putting his mental model triangles on the 95-mph (43 m/s) trajectory. Panel d shows the physical dimensions for adult baseball.

timating the speed of the pitch. For example, suppose that the pitcher threw a 95-mph (43 m/s) fastball, but the batter underestimated its speed and thought it was only going 90 mph (40 m/s). He would expect to hit the ball 3.46 ft (1.05 m) above the ground (from Column 7, 398-ms row of Table 1). But if he were to take his eye off the ball (indicated by the absence of some circles in the actual 95-mph [43 m/s] pitch of Figure 1b) and look at his bat to see the expected bat–ball collision, then the next time he saw the ball it would be 3.72 ft (1.13 m) above the ground (Column 3, 377-ms row of Table 1), 3 in. (7.62 cm) above his bat. He might explain this by saying, "The ball jumped three inches right in front of the plate." Should the batter underestimate the speed at 75 mph (34 m/s), he would say the ball jumped a foot (0.30 m), although the manager in the dugout, the catcher, and the center field TV camera, having different perspectives, would not see this jump.

Batters use one of two strategies in tracking the pitch (Bahill & LaRitz, 1984). The optimal learning strategy, which allows the batter to see the ball hit the bat, is the following: Track the ball over the first part of its trajectory with smooth pursuit eye movements, make a saccadic eye movement to a predicted point ahead of the ball, continue to follow the ball with peripheral vision letting the ball catch up to the eye, and finally, at the end of the ball's flight, resume smooth pursuit tracking with the images of the ball and bat on the fovea. It is called the *optimal learning strategy* because the batter observes the ball, makes a prediction of where it will cross the plate, sees the actual position of the ball when it crosses the plate, and uses this feedback to learn to predict better the next time the pitcher throws that pitch. The optimal hitting strategy, which does not allow the batter to see the ball hit the bat, is the following: Track the ball with smooth pursuit eye movements and fall behind in the last 5 ft (1.52 m). It is called the *optimal hitting strategy* because the batter keeps his eye on the ball longer, which should allow him to make a more accurate prediction of where the ball will cross the plate. We have no

Table 1
Comparison of Various Pitches

Time since release	Fastball 95 mph x	z	φ	Fastball 90 mph x	z	φ	Curveball 80 mph x	z	φ	Curveball 75 mph x	z	φ
0	55.5	6.00	0.00	55.5	6.00	0.00	55.5	6.00	0.00	55.5	6.00	0.00
50	48.5	5.96	−0.05	48.9	5.96	−0.05	49.6	6.15	0.17	50.0	6.13	0.16
100	41.6	5.84	−0.23	42.3	5.84	−0.23	43.8	6.18	0.24	44.5	6.16	0.21
150	34.6	5.64	−0.63	35.7	5.64	−0.61	37.9	6.09	0.14	39.0	6.06	0.10
200	27.6	5.36	−1.41	29.1	5.36	−1.34	32.0	5.89	−0.21	33.6	5.86	−0.26
250	20.7	4.99	−3.00	22.5	4.99	−2.74	26.2	5.57	−1.00	28.0	5.53	−1.00
300	13.7	4.55	−6.77	15.9	4.55	−5.74	20.3	5.14	−2.63	22.5	5.10	−2.45
350	6.7	4.03	−20.63	9.3	4.03	−14.18	14.4	4.59	−6.23	17.0	4.55	−5.33
377	3.0	3.72	−56.70									
398				3.0	3.46	−59.48						
400							8.6	3.92	−16.35	11.5	3.89	−11.90
448							3.0	3.17	−62.04			
450										6.0	3.11	−32.53
478										3.0	2.63	−65.98

Note. x is the distance to the tip of the plate. z is the height of the ball above the ground. φ is the angle of the batter's line of sight to the ball with respect to horizontal. x and z values are in feet; φ values are in degrees. Convert miles per hour to meters per second by multiplying miles per hour by 0.447. Convert feet to meters by dividing the value by 3.28.

evidence that batters voluntarily switch between these two strategies.

With the optimal learning strategy a batter would perceive more rising fastballs because his eye would not be on the ball (which is why we removed some circles from Figure 1b) when he started his swing, which occurs 100–150 ms before bat–ball contact when the ball is about 20 ft (6.1 m) in front of the plate, and therefore during the pitch the batter could not discover inaccuracies in his estimation of pitch speed and make adjustments. Whereas with the optimal hitting strategy, a batter would perceive fewer rising fastballs because his eye would be on the ball when it was 15–25 ft (4.57–7.62 m) from the plate; he could therefore sense inaccuracies in his speed estimation and, as shown in the last 150 ms of Figure 1c, make appropriate corrections.

McBeath (1990) did not have a quantitative model to investigate the perceptual illusion of the rising fastball. In the next section we develop such a model based on Todd (1981) and Regan (1986). This model uses only experimentally verified primary visual processes.

The Quantitative Model for the Rising Fastball

The batter must predict precisely where the ball will be in three-dimensional space at some specific future instant. This judgment involves four coordinates (the x, y, and z spatial coordinates shown in Figure 2, and t, the time coordinate). It is important to note that the batter has no direct visual access to the x, y, and z spatial coordinates: His judgments must be based entirely on retinal image data. The relevant retinal parameters are the angular size of the ball, γ, and the angular distance of the ball's image off the fovea, Φ, shown in Figure 2, and their time derivatives $d\gamma/dt$ and $d\Phi/dt$. We assume that the batter's eyes are in the x–y plane as shown in Figure 2, although the Pittsburgh Pirate studied by Bahill and LaRitz (1984) actually rotated his head 23° in pitch and 12° in roll.

Predicting When

To hit the ball, the batter must predict when and where it will cross the plate. First, let us see how the batter can judge *when.* In his novel *The Black Cloud*, Sir Fred Hoyle (1957) showed that the time to contact (τ) with an object moving along the line of sight can be approximated with

$$\tau = \frac{\gamma}{d\gamma/dt}, \tag{1}$$

where γ and $d\gamma/dt$ are the object's angular size and rate of change of angular size, respectively. It has been suggested that birds use time to contact when diving into the water to catch prey, and athletes use time to contact when jumping to hit a dropped ball, adjusting strides when running hurdles, and timing their swings in table tennis; for these tasks, time to contact is judged with an accuracy of about 2–10 ms (Bootsma & van Wieringen, 1990; Lee, Young, Reddish, Lough, & Clayton, 1983). Cricket players time their swings with an accuracy of ±2 ms (Regan, 1992). The batter's calculation of time to contact has three sources of error. First, Equation 1 is only an approximation because it uses the approximation $\tan\Psi = \Psi$. Second, the ball is not headed directly at the batter's eye. In our simulations, these two sources produced errors of less than 1 ms until the ball was within 150 ms of the contact point. The third source of error, which results from the batter hitting the ball 1.5 ft (0.46 m) in front of his eyes, produces a constant 11 ms of error.

The human visual system can implement Equation 1. First, there is psychophysical evidence that the human brain contains units tuned to size (γ), and size-tuned neurons have been found in monkey visual cortex. Second, psychological studies have shown that the visual system has specialized "looming detectors" that compute $d\gamma/dt$ independently of the object's trajectory (Regan & Beverley, 1978, 1980). Furthermore, specific brain neurons are sensitive to changing size ($d\gamma/dt$) (Regan & Cynader, 1979; Saito et al., 1986). By using these two pools of neurons, the brain could compute τ.

Is the movement of the baseball within physiological thresholds? For objects subtending less than 1.5°, $d\gamma/dt$ as low as 0.02°/s can be detected (Regan & Beverley, 1978). Lee (1976) suggested a threshold of 0.08°/s. When the pitcher releases a 95-mph (43 m/s) fastball, γ is ¼°, and $d\gamma/dt$ is 0.66°/s. Therefore $d\gamma/dt$ is well above visual threshold from the moment the ball leaves the pitcher's hand. The value of γ remains below 1.5° until the ball is 10 ft (3.05 m) from the tip of the plate. Figure 3 shows how γ and $d\gamma/dt$ change during the flight of the ball. The arrow shows where γ crosses its thresholds.

We conclude that from the instant the ball leaves the pitcher's hand, the batter's retinal image contains accurate cues for time to contact and that the human visual system is capable of using these cues. Evidently most batters estimate

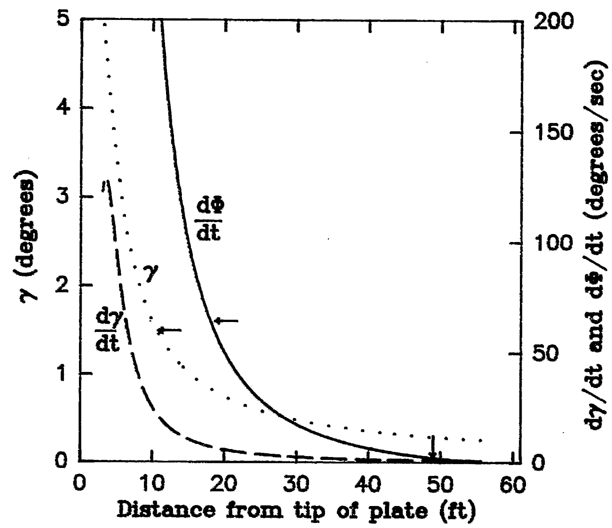

Figure 3. Physiological clues to ball position, γ, $d\gamma/dt$, and $d\Phi/dt$, as functions of distance to the tip of the plate, for a 95-mile-per-hour (43 m/s) fastball.

when the ball crosses the plate with an accuracy of at least ±9 ms, otherwise the ball would be hit foul.[1] Next, we want to consider the more difficult issue of judging where the ball will be at the time of bat–ball contact.

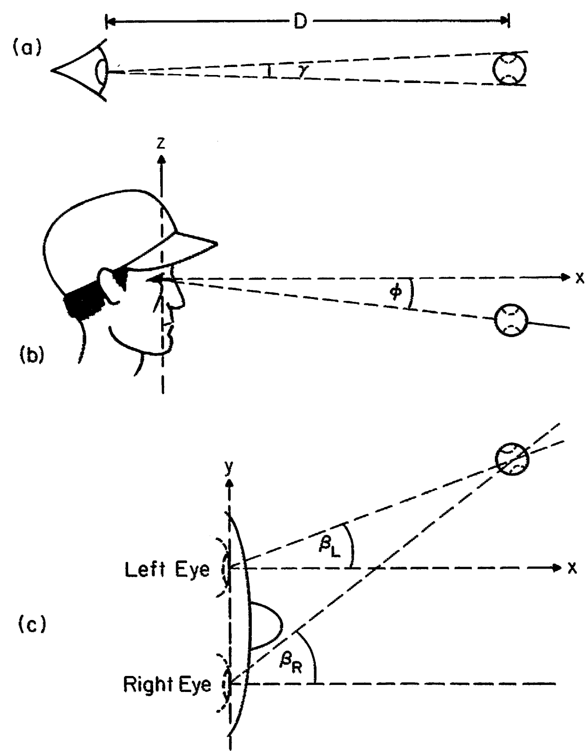

Figure 2. Visual system parameters used by the batter. (Panel a shows the angular size of the baseball, γ; the batter has no means to sense the distance to the ball [D]. Panel b shows the distance of the ball's image off the batter's fovea, Φ. Panel c shows the horizontal angle of the right and left eyes, β_R and β_L. *Note.* From "A Model for the Rising Fastball and Breaking Curve Ball" by W. J. Karnavas, A. T. Bahill, and D. Regan, 1990, *Proceedings of 1990 IEEE International Conference on Systems, Man, and Cybernetics*, p. 925. Copyright 1990 by The Institute of Electrical and Electronics Engineers. Reprinted by permission.)

Predicting Where

The batter can estimate the ball's height at contact from the ball's vertical speed and the time to contact. He can estimate the ball's vertical speed from the retinal velocity and the distance to the ball. The smallest value of retinal velocity, $d\Phi/dt$, that can be detected is about 0.02°/s. Over a range of moderate speeds (between about 2°/s and 64°/s) the discrimination of differences in $d\Phi/dt$ is within 5% (McKee, 1981; Orban, De Wolf, & Maes, 1984). Figure 3 shows that for a 95-mph (43 m/s) pitch, the value of $d\Phi/dt$ reaches 0.02°/s before the ball has traveled 1 ft (0.30 m), reaches 2°/s 49 ft (14.94 m) from the tip of the plate (vertical arrow), and reaches 64°/s 18 ft (5.49 m) from the tip of the plate (horizontal arrow). Therefore, the batter has an accurate sense of $d\Phi/dt$ that diminishes only after the swing has begun. (The swing starts when the ball is about 20 ft [6.1 m] away.)

Should we use φ, as defined in Figure 1, or Φ, as defined in Figure 2? In the last paragraph we used ($d\Phi/dt$), the

[1] The ball can be hit into fair territory despite errors in the angle of the bat of up to ±23°. If we assume that the bat rotates around a point between the two fists, this translates into errors of ±10 in. (25.4 cm). (If we considered rotations around the batter's body, this number would be reduced.) For a 90-mph (40 m/s) pitch, this maps into errors of ±9 ms, which is similar to times that can be derived from the angular velocity data of Messier and Owen (1984). Baseball batters probably need an accuracy much greater than ±9 ms because cricket batters need a timing accuracy of ±2 ms (Regan, 1992).

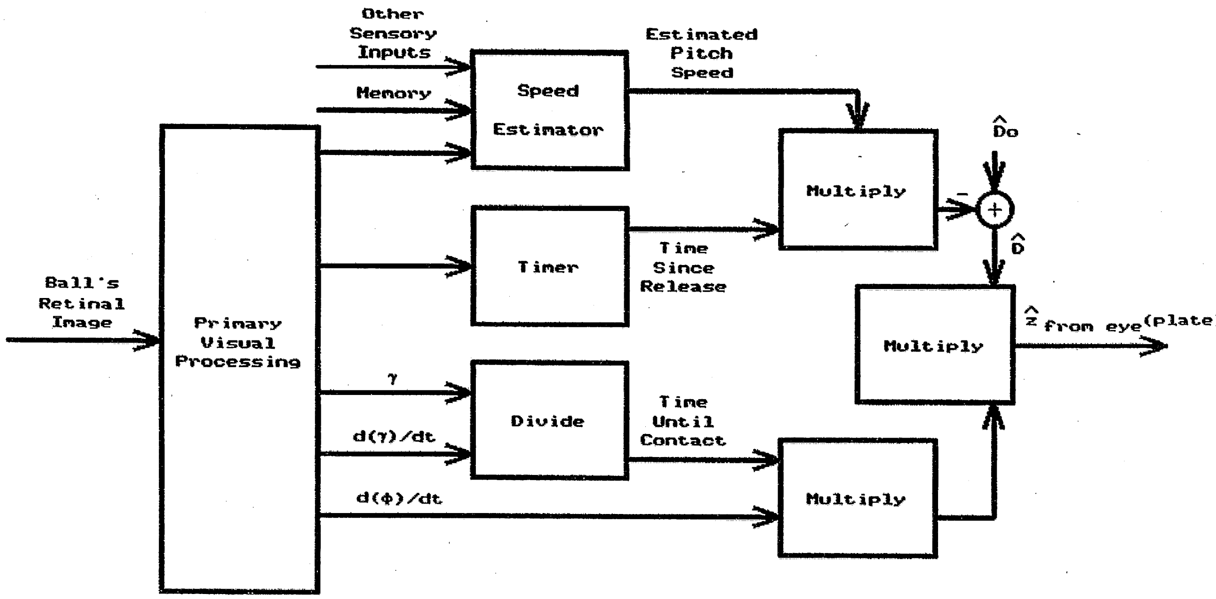

Figure 4. A model for a batter trying to estimate the height of the ball when it crosses the plate. (The input to the system is the ball's retinal image, and its output is the estimated height of the ball when it crosses the front edge of the plate.)

retinal image speed. This presumes that the batter's eye is stationary and that the ball's image moves across the retina. However, baseball batters use smooth pursuit eye movements to track the ball (Bahill & LaRitz, 1984). So perhaps we should use $d\varphi/dt$, eye velocity. For most humans, smooth pursuit eye movements are also accurate to within 5% for speeds up to 64°/s (Bahill & LaRitz, 1984; Bahill & McDonald, 1983b). So it does not make much difference whether we use φ or Φ in the model.

Now comes the crucial element in our explanation of the illusion of the rising fastball. Although retinal image information provides an accurate cue for the ball's time to contact, it provides poor cues to the absolute distance to the ball (D in Figure 2a) and for its line-of-sight speed. Classical stereoscopic depth perception is of little help in this regard; although the stereoscopic depth system provides a precise indication of relative depth (i.e., the difference between the x-axis distances of two objects imaged near the fovea), it provides little indication of absolute distance. In tracking the pitched ball, the batter has one object, the ball, imaged on his fovea. Therefore, the batter cannot measure distance to the ball or pitch speed; he can only estimate them.

Our psychophysical explanation for the rising fastball is as follows. The batter can estimate pitch speed and the time since the ball left the pitcher's hand. He can use this data in conjunction with his experience to estimate the distance to the ball ($\hat{D}$). He can then use this estimate for distance and the ball's retinal image velocity ($d\Phi/dt$) to estimate the vertical velocity ($d\hat{z}/dt$). From the vertical velocity and the time to contact (τ) he can estimate how far the ball will fall in the last part of its flight and therefore estimate the height of the ball when it crosses the front edge of the plate, $\hat{z}$(plate).[2] For example, suppose that the pitcher enhances the batter's expectations with a series of 90-mph (40 m/s) pitches and then throws an unusually fast 95-mph (43 m/s) pitch. Assume that the batter uses a 90-mph (40 m/s) mental model to interpret the retinal image information about the 95-mph (43 m/s) pitch. Suppose that the batter tried to estimate the ball's vertical speed 250 ms after the ball left the pitcher's hand. If the actual pitch were a 95-mph (43 m/s) fastball, the ball would be 19.2 ft (5.85 m) from his eye, subtracting 1.5 ft (0.46 m) (the distance between the tip of the plate and the batter's eye) from the x distance of 20.7 in Table 1. Its vertical velocity of −8 ft/s (−2.44 m/s) (from velocity = gt) would, at this distance, produce a retinal velocity of about −46°/s. However, if the batter thought the pitch was a 90-mph (40 m/s) fastball, he would think that it was 21 ft (6.4 m) away 250 ms after release. At this distance, a retinal image velocity of −46°/s would indicate that the vertical velocity was about −10.2 ft/s (−3.11 m/s). So the batter would think the ball was falling faster than it really was. Therefore, if the batter made a saccadic eye movement to a predicted point ahead of the ball, this point would be below the ball when the ball caught up with the eye, and the ball would seem to have jumped upward, in this example by more than 2 in. (5.08 cm). To be sure, this error of visual judgment could be avoided if the batter had an accurate visual cue to the ball's absolute distance (D) or its speed, but as we have seen, the batter is essentially "blind" to these two important parameters.

[2] The vertical velocity is not constant, so this is only an approximation. We will examine the consequences of this approximation in a later section.

This model can be summarized by Figure 4 and

$$\hat{z}_{\text{from eye}}(\text{plate}) = (\hat{D}_0 - \hat{t}_{\text{sr}}\hat{s})\left(\frac{d\Phi}{dt}\frac{\gamma}{d\gamma/dt}\right), \qquad (2)$$

where $\hat{z}_{\text{from eye}}(\text{plate})$ is the estimated vertical distance from the batter's eye when the ball crosses the front edge of the plate, $\hat{D}_0$ is the estimated distance between the ball and the batter's eye at the time of release, $\hat{t}_{\text{sr}}$ is the estimated time since release, $\hat{s}$ is the estimated pitch speed, $d\Phi/dt$ is the retinal image velocity, γ is the retinal image size, and $d\gamma/dt$ is the rate of change of retinal image size. To derive this equation, we have taken advantage of the approximation $\tan\Psi = \Psi$, where Ψ is any small angle. The first term of Equation 2 is the estimated distance to the ball, $\hat{D}$, and the last term is τ from Equation 1, so Equation 2 can be rewritten as

$$\hat{z}_{\text{from eye}}(\text{plate}) = \hat{D}\frac{d\Phi}{dt}\tau. \qquad (3)$$

Overestimating any one of these three terms could produce the illusion of the rising fastball. But as we have already shown, batters can accurately estimate $d\Phi/dt$ and τ; the difficulty is in estimating the distance to the ball. And in this model batters use estimated pitch speed to estimate the distance to the ball. That makes the speed estimator in Figure 4 the crucial element of this model. To estimate the height of the ball at the time of contact, the batter must be able to estimate the pitch speed. But pitch speed cannot come from the primary visual processes. The speed estimator receives input from the primary visual processes, but this arrow is unlabeled because we do not know exactly which signals are involved. The speed estimator probably uses memory and other sensory inputs: some visual, such as the motion of the pitcher's arms and body, and some auditory, such as a whistle from a coach who might have stolen the catcher's signals.

The speed estimator might be most accurate just after the pitcher releases the ball, for at this point the batter should be able to estimate its distance $\hat{D}_0$ quite well. And the distance $\hat{D}_0$ combined with the time to contact would provide the speed. But at this time the estimation of the ball's height at the time of contact may still be in error, because retinal image velocity $d\Phi/dt$ is below threshold for the first foot and is inaccurate until the ball is 49 ft (14.94 m) from the tip of the plate, and eye velocity $d\varphi/dt$ was always zero until the ball was halfway to the plate (Bahill & LaRitz, 1984; Watts & Bahill, 1990).

Possible Variations

We considered many other signals that might help estimate the speed of the ball and its distance, but we discarded them because they were inappropriate. For example, the retinal image could contain a cue to line-of-sight speed (dD/dt) because $dD/dt = (D/\gamma)(d\gamma/dt)$. And, as we have already shown, the batter has visual clues for γ and $d\gamma/dt$, but unfortunately the batter has no reliable visual cue to absolute distance D, so he is unable to take advantage of the geometrical fact expressed in this equation. We considered many techniques for calculating the distance to the ball. (a) The vergence eye movement system (the difference between β_L and β_R in Figure 2c) cannot help, because vergence changes do not contribute to motion-in-depth sensation (Regan, Erkelens, & Collewijn, 1986). (b) The differences in the batter's lines of sight for the two pitches of Figure 1 do not differ by 1° until the ball is 6 ft (1.83 m) from the batter, and extraretinal sensation of eye position is only good to about 1° (Matin & Kibler, 1966; Steinbach, 1970). (c) A human can discriminate precisely the direction of motion in the horizontal plane to within 0.2° (Beverley & Regan, 1975; Regan, Beverley, & Cynader, 1979). However, our present study is about vertical motion and not horizontal motion, so this clue is irrelevant. (d) Parallax is often used to help judge distance. However, the batter starts his translational head movements in the last third of the ball's flight only (Bahill & LaRitz, 1984; Watts & Bahill, 1990), and this is too late to help him. (e) Apparent size is often used as a clue to distance. In fact, Todd's (1981) model gives the distance to the ball in terms of ball diameters. We will return to this clue in a later section of this article. (f) We have already commented that stereoscopic depth perception gives clues about the relative depth of two objects only and not about absolute distance to a single object, so it cannot help and is in fact not necessary for good hitting. Indeed, according to Kara (1990), an ophthalmologist who examined him, Babe Ruth was amblyopic and never had more than 20/200 vision in his left eye. Therefore, we conclude once again that the batter has no information to determine accurately the ball's absolute distance (D) from his eye.

We have said before that the batter must predict both where and when the bat–ball collision will occur. To hit a line drive in fair territory, the batter must estimate *when* to within ±9 ms and *where* to within ±½ in. (1.27 cm) (Watts & Bahill, 1990). Comparing these numbers to the 21 ms and 3.1-in. (7.87-cm) differences between the 90- and 95-mph (40 and 43 m/s) fastballs, we see that estimating *where* requires a greater percentage accuracy. And batters seem to be more accurate at estimating *when*. Indeed, few line drives are hit in foul territory, whereas there are many foul tips and popups. Figure 4 suggests an explanation: *when*, τ, can be computed from primary visual processes, whereas the pitch speed, and therefore *where*, can only be estimated. So *where* is the crucial parameter: It requires greater precision, yet it cannot be calculated from primary visual processes; it can only be estimated.

Although we developed this model to explain the illusion of the rising fastball, it could of course also be applied to other pitches. For example, the change-up is effective only if it fools the batter and makes him overestimate the pitch speed. According to our model, this would make the batter swing over the ball. Although we have not heard coaches describe it this way, our model predicts that an effective change-up should result in many ground balls to the third baseman.

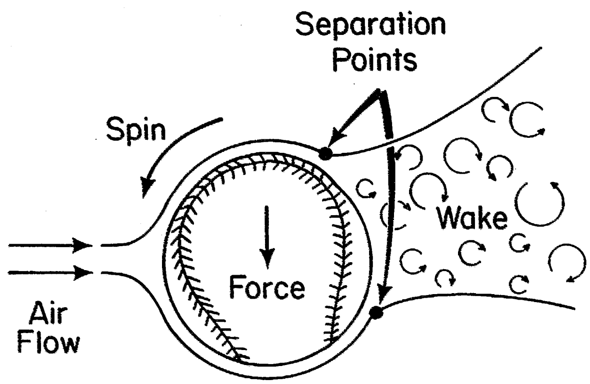

Figure 6. Air flows smoothly around the ball until it gets to the separation points. There the air flow changes into a chaotic, swirling flow called the *wake*. If this were a top view, it would explain the curve of a ball. If it were a side view, it would explain the abrupt drop of the ball. *Note.* Reprinted from *Engineering Modeling and Design* (p. 33) by W. Chapman, A. Bahill, and W. Wymore, 1992, Boca Raton, FL: CRC Press. Copyright CRC Press. Reprinted by permission.

You can feel this force if you put your hand out the window of a moving car. (Make sure the driver knows you are doing this!) Tilt your hand so that the wind hits the palm of your hand at an angle. This deflects the air downward, which causes your hand to be pushed upward. Now let us relate this to the spinning baseball in Figure 6. Before the ball interacts with the air, all the momentum is horizontal. Afterward, the air in the wake has upward momentum. The principle of conservation of momentum therefore requires that the ball have downward momentum. Therefore, the ball will move downward.

There are several ways to shift the wake behind a baseball. The wake is shifted by the spin on a curveball. The friction that slows the flow of air over the top of the ball causes the air to separate from the ball sooner on the top than on the bottom, as shown in Figure 6. This shifts the wake upward, thus pushing the ball downward. For nonspinning pitches such as the knuckleball and the scuff ball, when the seams or the scuff are near the bottom separation point they create turbulence, which delays the separation, as shown in the bottom of Figure 6, again shifting the wake upward and pushing the ball downward.

So, when the pitcher puts spin on the ball, the wake of chaotic air behind the ball is moved to one side, causing the ball to curve and thereby confounding the poor batter who is trying to hit it.

The Curveball Simulation

Now let us return to the breaking curveball. The simplified 90-mph (40 m/s) pitch of Table 1 and Figure 1 falls 2½ ft (0.76 m) because of gravity. A plot of this vertical distance as a function of time would be parabolic ($\frac{1}{2}at^2$). The ball falls 19 in. (48.26 cm) in the 150 ms right before contact but only 10 in. (25.4 cm) in the 150 ms before that. The ball drops more in the last 150 ms than in the period right before it, but it follows a smooth parabolic trajectory. Now let us see how the ball drops with the addition of a vertical force due to spin on the ball. The right side of Table 1 shows the results of simulations of 80- and 75-mph (36 and 34 m/s) curveballs. Both were launched at an angle of 2° up with 1,900 rpm of pure topspin. We used the following formula from Watts and Bahill (1990) for the downward force due to spin: $F = \frac{1}{2}\rho\pi R^3\omega v$, where ρ is the air density, R is the radius of the baseball, ω is its spin rate, and v is its velocity. Now let us look at the column in Table 1 for the 80-mph (36 m/s) pitch. We can see that the ball falls 24 in. (60.96 cm) in the 150 ms right before contact, but only 11 in. (27.94 cm) in the preceding 150 ms. Once again the ball drops more in the last 150 ms than in the earlier epoch, but it still follows a parabolic trajectory. Therefore, this is a curve, not a break. A breaking curveball would have to drop the same amount as this ball in the early 150-ms epoch, but more than this in the last 150 ms. It would be impossible to hit the ball if it really did jump or break just before the plate. For example, a foul tip changes the ball's trajectory abruptly in the last few feet before the catcher and the umpire. As a result, these masked men cannot predict the ball's trajectory, and they must wear protective clothing.

The Breaking Curveball Illusion

We are now ready to explain the apparent abrupt break of some curveballs. Suppose that the pitcher threw the 75-mph (34 m/s) curveball of Table 1; it would drop 27 in. (68.58 cm) in the last 150 ms before contact. However, if the batter overestimated the pitch speed and thought it was going 80 mph (36 m/s), then he would expect it to fall 24 in. (60.96 cm) in the last 150 ms before contact. Thus if he took his eye off the ball 150 ms before the projected time of contact and saw it again when it arrived at his bat, he would think that it broke down 3 in. (7.62 cm). Therefore, we suggest that the apparent abrupt break of some curveballs might be a result of the batter's *overestimation* of the speed of the pitch in his mental model.

Of course, the illusion would be greatly enhanced if the batter made a mistake in estimating both the speed and the type of the pitch. If the pitch were a 95-mph (43 m/s) fastball with 1,600 rpm of backspin and the batter thought it was a 75-mph (34 m/s) curveball with 1,900 rpm of topspin, then the illusionary jump would be 1 ft (0.30 m). Similarly, if such a curveball were mistaken for such a fastball, then the illusionary break would be 1 ft (0.30 m).

In summary, we think that every pitch (except a nonspinning pitch such as the knuckleball) follows a smooth trajectory (Watts & Bahill, 1990). The apparent abrupt jumps and drops right before the plate are perceptual illusions caused by the batter using the optimal learning strategy of tracking and making mistakes in his mental model of the pitch. We think that in the first third of the ball's flight, the batter forms his mental model of the pitch. In the middle third, he observes differences between the actual trajectory and his mental model, and updates his mental model. Then he starts his swing. In the last third of the ball's flight, the batter either observes errors in his mental model so that he can track the next pitch better (the optimal learning strategy)

Experimental Validation of the Model

To help validate our model, we ran some simple experiments with a mechanical pitching machine (the two-wheeled type). We threw 450 pitches to 7 subjects, 3 adults and 4 boys aged 9, 11, 11, and 13. Nominally the speed of the machine was set for 50 mph (22 m/s), but occasionally it threw a fast pitch at 55 mph (25 m/s). The number of normal pitches between these fast pitches was randomly chosen from among 3, 4, 5, and 6. An observer (who did not know the pattern of normal and fast pitches) recorded the relationship of the bat and ball when the ball crossed the plate. We averaged the results of the fast pitches and of the two pitches before and after, as shown in Figure 5. These data show that on the fast pitches the batters swung below the ball, which is just what would happen if they underestimated the speed of the pitch.

The pitching machine was not perfect. About 15% of the time, it threw "balls" out of the strike zone. This randomness lessened our batters' expectations, and therefore should have lessened the effect of the rising fastball. Most of our batters "took" (did not swing at) these "balls." However, they seldom took a strike. Except, and much to our surprise, they often took the unusually fast pitches even when they were in the strike zone. We recorded these called strikes and indicate their number with the Cs in Figure 5. They might have taken these "called strikes" because they underestimated the pitch speed and were therefore going to swing under the ball; but in the last third of its flight, when it was too late to alter the swing, they realized their error and did the only thing they could do, stop the swing. These called strikes also decreased the effects of the rising fastball in our data, because our batters did not swing at the pitches that fooled them the most, the pitches that would have shown the greatest effects. The adults in our study understood the purpose of the study and swung at every pitch. It was only the boys, who did not understand the purpose of our experiments, who took the pitches.

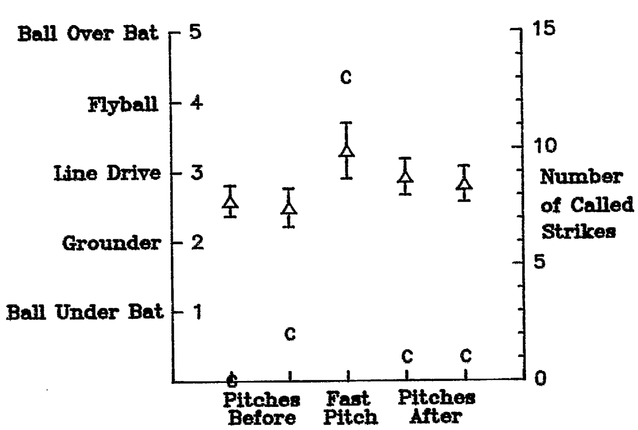

Figure 5. Averaged data from 7 batters showing that when an unusually fast pitch was thrown most batters swung under the ball or "took a called strike." (The capital Cs represent the number of called strikes. The triangles are the mean values, and the vertical bars are the 95% confidence intervals.)

The data shown in Figure 5 are the averages of all our subjects. Each of our 7 individual subjects showed this same effect, but it was more emphatic in our inexperienced subjects.

So, in summary, both the randomness of our pitching machine and the batters taking called strikes decreased the effect of the rising fastball in our data. But in spite of these imperfections, it was still statistically significant that when the unusually fast pitch was thrown, the batters swung under the ball. This is precisely the illusion of the rising fastball.

The Breaking Curveball

To test the model further, we used it in a situation for which it was not designed. We asked what would happen if the batter overestimated the speed of the pitch? He would perceive the ball dropping more than expected in the last few feet. The ball would break so fast it would look "like it rolled off a table." Such fast-breaking curves have been described by professional batters. However, the laws of physics say that the curveball must exhibit a continuous curve and not an abrupt break near the plate (Watts & Bahill, 1990). We will use our model to help explain the illusion of the breaking curveball, but first we must explain the simple curveball.

Why Does the Curveball Curve?

There is no longer a controversy about whether a curveball curves; it does. The curveball obeys the laws of physics. These laws say that the spin of the ball causes the curve (Watts & Bahill, 1990). Should this spin be horizontal (as on a toy top), the ball curves horizontally. If it is topspin, the ball drops more than it would due to gravity alone. If it is somewhere in between, the ball both curves and drops. In baseball, most curveballs curve horizontally and drop vertically. The advantage of the drop is that the sweet spot on the bat is about 6 in. (15.24 cm) long but only ½ in. (1.27 cm) high. Thus a vertical drop would be more effective at taking the ball away from the bat's sweet spot than a horizontal curve. We now present the principles of physics that explain why the curveball curves.

The first part of our explanation invokes Bernoulli's principle. When a spinning ball is placed in moving air, as shown in Figure 6, the movement of the surface of the ball and a thin layer of air that "sticks" to it slows down the air flowing over the top of the ball and speeds up the air flowing underneath the ball. Now, according to Bernoulli's equation, the point with lower speed (the top) has higher pressure, and the point with higher speed (the bottom) has lower pressure. This difference in pressure pushes the ball downward.

The second (and probably more important) part of our explanation involves the wake of chaotic air behind the ball. You can see such a wake on the downstream side of bridge abutments and behind boats. In a boat, swinging the rudder to the right deflects water to the right, and to conserve momentum the back of the boat must be pushed to the left.

air resistance has little effect on our conclusions about the illusion of the rising fastball.

Baseball players describe the rising fastball and the breaking curveball, but oddly enough, they do not talk about a dropping fastball or a rising curveball. According to our simulations and models, if the pitcher threw a string of 90-mph (40 m/s) fastballs and followed up with an 85-mph (38 m/s) fastball, and the batter took his eye off the ball when it was 150 ms before the contact point, then the batter might think it dropped more than normal. Perhaps when this occurs batters merely think the pitch was a fast curveball.

From these studies, we can see that estimating pitch speed is important for the batter. It seems that a useful training technique would be to use a radar gun during batting practice and announce the speed immediately after every pitch. This would help the batter learn to estimate pitch speed better.

Enhancements to the Model

The model of Figure 4 is based on psychological and physiological principles. We cited human psychological experiments that showed that humans use each of the signals used in this model. Furthermore, we cited physiological experiments that showed that pools of neurons in the brains of vertebrates respond to these same signals.

However, earlier we said that the model of Figure 4 only yields an approximation for height at contact because the model multiplies the ball's vertical velocity by the time to contact to estimate how far the ball will fall in the last part of its flight. The ball's actual vertical velocity is not constant over this interval. To illustrate the consequences of this simplification, note that 250 ms after the simplified 90-mph (40 m/s) fastball is released, the model of Figure 4 estimates its height at contact as 3.71 ft (1.13 m). However, if we consider the increasing vertical velocity due to the acceleration of gravity, we find that the actual height at contact is 3.46 ft (1.05 m), yielding an error of 3 in. (7.62 cm). Baseball players seldom make such large errors, so they must be using other information, perhaps information about the ball's vertical acceleration.

To take vertical acceleration into account, we can modify the model of Figure 4 by using vertical acceleration times time in the bottom multiplier box. We will call this the *second-order Model 1*. However, acceleration is not available from primary visual processes. Therefore, acceleration must be estimated, perhaps from time since release, time to contact, and vertical velocity. Calculating vertical acceleration with these parameters requires knowledge of the force of gravity, the spin on the ball, and the launch angle. The first element, the force of gravity, is a constant, so it can be learned (Todd & Bressan, 1990). For the second element, batters say it is essential to detect the spin on the ball; perhaps they use their knowledge about spin to help estimate vertical acceleration. Concerning the third element, for our simple fastballs of Table 1 we used a launch angle of zero. In reality, this angle could range from −4° for a professional fastball to 12° for a Little League curveball. This angle would have to be perceived and used to help estimate vertical acceleration. Acceleration is an important parameter in determining what visual waveforms humans can learn to track (Bahill & McDonald, 1983a, 1983b). However, humans estimate acceleration due to spin on a baseball poorly (Todd, 1981). In general, "the human visual system is ill equipped to compute second-order temporal derivatives" (Todd & Bressan, 1990).

Figure 7 shows the estimated vertical distance from the eye when the ball reaches the front edge of the plate as a function of where the estimation was made. The horizontal lines represent perfect prediction for the 95-mph (43 m/s) fastball of Table 1 (solid lines) and the 80-mph (36 m/s) curveball of Table 1 (dashed lines). The lines sweeping from the upper right to the lower left show the output of the first-order model (without acceleration), that is, Equation 2. This model is most accurate when the ball is 15–20 ft (4.57–6.1 m) from the plate, which coincidentally is the ball's position when the batter starts his swing. The first-order model is most accurate at this position for all pitches we simulated (i.e., fastballs, curveballs, change-ups, etc.). By adding acceleration to Equation 2, we get

$$\hat{z}_{\text{from eye}}(\text{plate}) = \tfrac{1}{2}(\hat{D}_0 - \hat{t}_{sr}\hat{s})\tau^2 \frac{d^2\Phi}{dt^2}, \quad (4)$$

which is our *second-order Model 1*. The other lines in Figure 7 show the output of this second-order model. The error between the second-order model and the ideal prediction is due to (a) computational noise, (b) the ball not heading directly at the batter's head, and (c) the batter hitting the ball 1.5 ft (0.46 m) in front of his eyes.

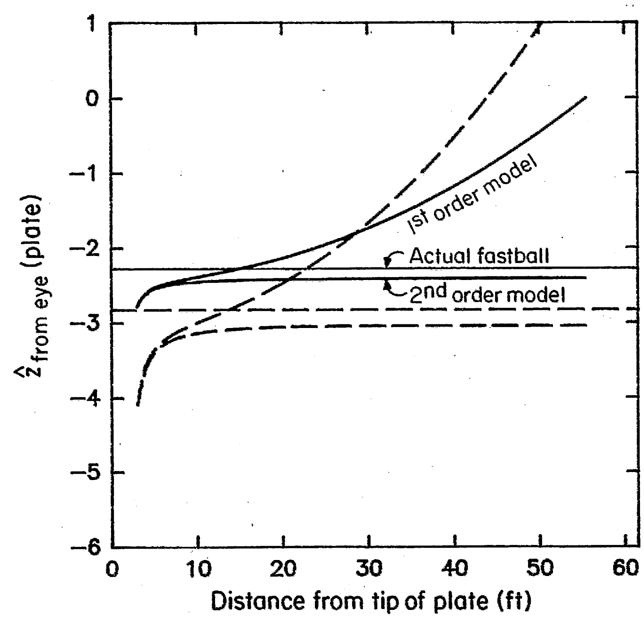

Figure 7. Estimated vertical distance from the batter's eye when the ball reaches the front edge of the plate plotted as a function of where the ball was when the estimate was made. (Perfect prediction and the results of the first- and second-order models are shown for a simplified fastball [solid lines] and a curveball [dashed lines]).

Table 2
Results of Sensitivity Analyses

Parameter	Value
Sensitivity coefficients for the simulation	
Horizontal position of the pitcher's release point	−0.2
Vertical position of the pitcher's release point	0.0
Launch angle	0.0
Horizontal position of the batter's eye	0.0
Vertical position of the batter's eye	0.0
Horizontal position of the bat–ball collision	−0.1
Pitch speed	18.9
Estimated pitch speed	−21.1
Sensitivity coefficients for the model	
$\hat{D}_O$, distance to release point	−2.57
$\hat{t}_{sr}$, time since release	1.57
$\hat{s}$, estimated pitch speed	1.57
$d\Phi/dt$, retinal image velocity	−1.00
γ, retinal image size	−1.00
$d\gamma/dt$, rate of change of retinal image size	0.95

or updates his mental model and possibly modifies his swing (the optimal hitting strategy).[3]

The Sensitivity Analyses

To help validate our simulation and model, we will now investigate how well our conclusions hold up under parametric sensitivity analyses. First we will show a sensitivity analysis of our simulation (Figure 1 and Table 1), and then we will show a sensitivity analysis of our model (Figure 4 and Equation 2).

Each of the eight simulation parameters was varied by ±5%, except the launch angle was changed from 0° to ±0.1° (a 2° variation in launch angle moves the ball from the top to the bottom of the strike zone, and 5% of 2° is 0.1°). We then ran the simulation and calculated the percentage change for the perceived jump. The ratio of these two percentages is the relative sensitivity (Bahill, 1981). These sensitivity values are shown in the top of Table 2.

The 5% perturbations produced similar results to the −5% perturbations. For simplicity, we list only the largest sensitivity coefficients, those that occurred for the −5% perturbations. To help understand these numbers, note that from our nominal simulation of Table 1 the apparent jump of the rising fastball is 0.2608 ft (7.95 cm). When the horizontal position of the bat–ball collision point was moved back 5%, the amount of perceived jump increased to 0.2623 ft (7.99 cm), or by 0.57%, which (given roundoff errors) is −0.1 (the value of the sensitivity coefficient from Table 2) × 5% (the perturbation size). Similarly, when the estimated pitch speed was decreased 5%, the amount of perceived jump increased to 0.5357 ft (16.33 cm), or by 105.4%, which is 21.1 × 5%. Therefore, the simulation is sensitive to pitch speed and the estimated pitch speed; in comparison, all other parameters are insignificant.

We have also calculated sensitivity coefficients, as functions of the eight simulation parameters, for the distances at which the physiological parameters crossed their thresholds, as shown in Figure 3. The maximum values of these sensitivity coefficients were 0.1 for where $\gamma > 1.5°$, 1.2 for where $d\Phi/dt > 2°/s$, and 1.2 for where $d\Phi/dt > 64°/s$. This means that where the physiological parameters cross their thresholds is also insensitive to the eight simulation parameters.

Therefore, it is rewarding to see that the pitch speed and the estimated pitch speed are the most important parameters of the simulation. The sensitivity analysis tells us that we should not (a) try to get more accurate values for the other parameters of the simulation, (b) customize the simulation for individual players, or (c) try to get more accurate values for the thresholds of the visual parameters γ or $d\theta/dt$, because these parameters are not that important. The bottom line of this analysis is that our simulation is a good representation for the pitched baseball.

Next we did a sensitivity analysis of our model of Figure 4 and Equation 2 for a 90-mph (40 m/s) pitch. Each of the six model parameters was varied by ±5%, and we calculated the percentage change in how far the ball dropped between 250 ms after release and the time of bat–ball contact. The ratio of these two percentages is the relative sensitivity. These sensitivity values are shown in the bottom of Table 2.

The model is most sensitive to the three parameters that are used to estimate the distance to the ball. The batter cannot get these parameters from the primary visual processes. It seems that the model is most sensitive to the things that are most difficult for the human to compute. Perhaps superior athletes are superior not because they have superior primary visual processes, but because of their subsequent processing of this information.

Discussion

In our section on the rising fastball, for simplicity we did not include the effects of spin or air resistance. To see if these omissions effected our conclusions, we reran the simulation including both of these effects. We used the following equations from Watts and Bahill (1990): $F_{\text{lift}} = \frac{1}{2} \rho\pi R^3 \omega v$, and $F_{\text{drag}} = \frac{1}{4} \rho\pi R^2 v^2$, where ρ is the air density (typically 0.0023 lb s^2/ft^4), R is the radius of the baseball (typically 0.119 ft [3.63 cm]), ω is its spin rate (rad/s), and v is its velocity (ft/s). The 95-mph (43 m/s) fastball with 1,300 rpm of backspin was 4.34 ft (1.32 m) high moving at 83 mph (37 m/s) at the point of contact. The 90-mph (40 m/s) fastball with 1,300 rpm of backspin was 4.11 ft (1.25 m) high moving at 79 mph (35 m/s) at the point of contact. Therefore, if the batter underestimated the speed of the 95-mph (43 m/s) fastball by 5 mph (2.2 m/s) and took his eye off the ball 150 ms before contact, he would perceive an illusionary jump of 0.23 ft, or 2.8 in. (7.01 cm), a jump that is just about the same as shown earlier with the simpler simulation. Therefore the omission of the effects of spin and

[3] It might be impossible to do anything except check, or cancel, the swing in the last third of the ball's flight. Indeed, McLead (1987) has shown that cricket batters cannot alter their swings in the last 200 ms.

In summary, in the first part of this article we presented our first-order Model 1. It was based on sound physiological principles. However, it did not perform as well as humans do. So we expanded our model to a second-order Model 1. It performs almost as well as the human. But it is suspect because it uses accelerations, and humans do not use acceleration information well.

Bootsma's Model

Bootsma's model (e.g., see Bootsma & Peper, in press) for predicting the ball's position uses the optical variables $\bar{\gamma}$ and $\bar{z}$ defined in Figure 8. In equation form, his first-order model (Equation 23 of Bootsma & Peper, in press) is

$$\hat{z}_{\text{from eye}}(\text{plate}) = \frac{Bd\bar{z}/dt}{(d\bar{\gamma}/dt)}, \tag{5}$$

where $\hat{z}_{\text{from eye}}(\text{plate})$ is the estimated vertical distance from the batter's eye when the ball crosses the front edge of the plate, and B is the size of the ball. We will call this the *first-order Model 2*. If the human can estimate accelerations, then an appropriate second-order model (Equation 25 of Bootsma & Peper, in press) is

$$\hat{z}_{\text{from eye}}(\text{plate}) = \frac{B\bar{\gamma}d^2\bar{z}/dt^2}{2(d\bar{\gamma}/dt)^2}. \tag{6}$$

For tracking a pitched baseball, we can safely substitute Φ defined in Figure 2 for $\bar{z}$. With this substitution and an explicit representation for τ, we get our implementation of Bootsma's model:

$$\hat{z}_{\text{from eye}}(\text{plate}) = \frac{B_{\text{mm}}\tau d^2\bar{\Phi}/dt^2}{2(d\bar{\gamma}/dt)}, \tag{7}$$

where $\hat{z}_{\text{from eye}}(\text{plate})$ is the estimated vertical distance from the batter's eye when the ball crosses the front edge of the plate, B_{mm} is the batter's mental model of the size of the ball, and τ is the estimated time to contact from Equation 1. In effect, this model is giving the distance to the ball in terms of the batter's mental model of the size of the ball. We will call this the *second-order Model 2*. The results of running the first-order and second-order Model 2's are identical to those shown in Figure 7.

Comparing the Two Models

We will now compare Model 1, which is described by Figure 4 and Equations 2 and 4, with Model 2, which is described by Equations 5 and 7. The principal difference between the two models is that Model 1 uses time since release to help estimate the distance to the ball, whereas Model 2 gives distances in terms of the batter's mental model of the size of the ball. We will now make several comparisons between the performance of these two models. Do they accurately predict the height of the ball at the time of contact, presuming no errors in estimating ball speed or size? As shown in Figure 7, all of the models have some error. The predictions of the first-order Model 1 are identical to the predictions of the first-order Model 2. The predictions of the second-order model 1 are identical to the predictions of the second-order Model 2. The behavior of the two models is indistinguishable. Do they explain the illusion of the rising fastball? The first-order Model 1 does by assuming that the batter underestimates the pitch speed, and both of the second-order models can by assuming that the batter overestimates vertical acceleration. Do they explain experiments in which the speed of the pitch is secretly varied (e.g., Figure 5)? Model 1 does, but Model 2 does not. In several psychological experiments, the size of the ball has been secretly changed, and subsequently the subjects incorrectly predicted the position of the ball (Bootsma & Peper, in press; Savelsbergh, Whiting, & Bootsma, 1991). In particular, the data showed that when a smaller than normal ball was used, humans thought it would cross the plate higher than it actually did. Model 1 does not explain this behavior, but Model 2 does. Do they need to subtract distances (which may be difficult for humans)? Model 1 does ($\hat{D}_0 - \hat{t}_{sr}\hat{S}$), but Model 2 does not; everything is relative. Are they based only on known psysiological and psychological capabilities? Only the first-order Model 1 is. In conclusion, the jury

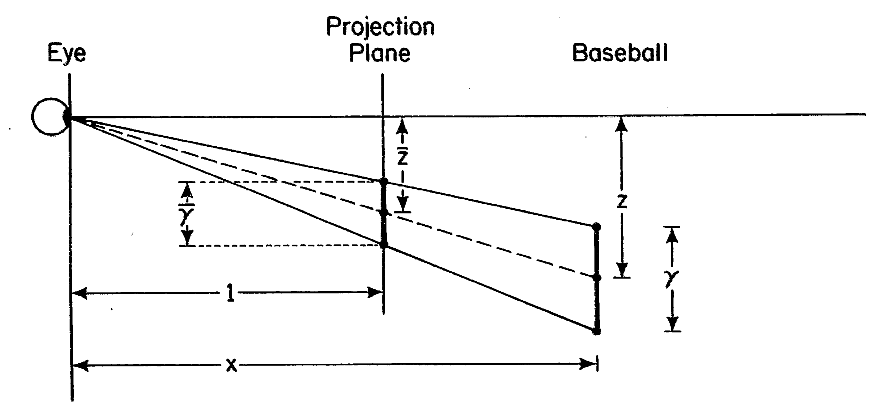

Figure 8. Definition of the optical variables. (γ is the annular size of the ball, z is its distance above the batter's eye, and x is its horizontal distance from the batter. $\bar{\gamma}$ and $\bar{z}$ are the annular size of the ball and its height projected onto a plane a unit distance away from the batter. Based on Todd [1981] and Bootsma and Peper [in press].)

is still out. Each model explains the data collected to verify it. We think independent experiments should be designed to test the two models directly. We hope that such data could be collected by using normal humans. However, if we are to assess the ultimate capabilities of humans, then perhaps we will have to use optimal humans. Unfortunately, this will cost science a lot of money, because a highly successful pitcher is paid $1,000 per pitch, and a highly successful batter is paid $1,000 per swing. Using professional athletes in such experiments may be expensive.

Summary

One by one, scientists and engineers, using principles of physics, have explained most of baseball's peculiar pitches, such as the knuckleball (Hollenberg, 1986; Watts & Sawyer, 1975), the scuff ball, and the curveball (Watts & Bahill, 1990). The most mysterious remaining pitches were the rising fastball and the breaking curveball. Now, adding principles of psychology, we can suggest that the rising fastball and the breaking curveball may be perceptual illusions caused by (a) the batter tracking the ball over the first part of its trajectory with smooth pursuit eye movements, making a saccadic eye movement to a predicted point ahead of the ball, and finally, at the end of the ball's flight, resuming smooth pursuit tracking, and (b) the batter misestimating the acceleration or speed of the pitch.

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Received October 18, 1990
Revision received January 22, 1992
Accepted January 22, 1992 ■