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Determining Ideal Baseball Bat Weights Using Muscle Force-Velocity Relationships

A. T. Bahill and W. J. Karnavas

Systems and Industrial Engineering, University of Arizona, Tucson, AZ 85721, USA

Abstract. The equations of physics for bat-ball collisions were coupled to the physiology of the muscle force-velocity relationship to compute the ideal bat weight for individual baseball players. The results of this coupling suggest that some batters use bats that are too heavy for them, and some batters use bats that are too light, but most experienced batters use bats that are just right. However, ideal bat weight is not correlated with height, weight, or age. Decades of prior physiological research on force-velocity relationships of isolated muscle have shown that hyperbolic curves usually fit the data best. However, for the present data, the hyperbolic curves fit only one class of subjects best: for the others a straight line provides the best fit. We hypothesize that these two classes of players use different control strategies.

Introduction

Over the last five decades units as small as isolated actin-myosin fibers (Cerven 1987) and as large as whole muscle (Wilke 1950) have been shown to obey Hill's force-velocity relationship (Fenn and Marsh 1935; Hill 1938). In this study we show that some human multi-joint movements are modeled best with Hill type hyperbolas, and some are modeled best with straight lines. The human multi-joint ballistic movement that we have chosen to study is that of a human swinging a baseball bat. We choose this particular movement because it is a commonly performed, stereotyped, multi-limb, ballistic movement that is performed often by many dedicated humans. Effects of sex, training and the time-varying characteristics of the force-velocity relationships can be easily controlled. It is safe and measurement is noninvasive. In a skilled practitioner it is repetitive and machine-like; there is little variability between successive swings. Furthermore it is possible to produce a set of bats that are matched in all respects except weight. This allows consistent data collection throughout the physiological range.

To find the best bat weight we must first examine the conservation of momentum equations for bat-ball collisions. As a simplifying assumption treat the bat-ball collision as linear: i.e. assume the ball and bat are both traveling in straight lines, as shown in Fig. 1. The principle of conservation of momentum says that

$$m_1 v_{1b} + m_2 v_{2b} = m_1 v_{1a} + m_2 v_{2a}.$$

The subscript 1 is for the ball, 2 is for the bat. The subscript b is for *b*efore the bat-ball collision, and a is for *a*fter the collision. Because the ball is moving to the left, v_{1b} is a negative number. For the science of baseball the distinction between mass and weight is not necessary, so we will substitute weight for mass in the above equation to produce

$$w_1 v_{1b} + w_2 v_{2b} = w_1 v_{1a} + w_2 v_{2a}. \tag{1}$$

Note, we are assuming that the mass of the batters arms has no effect on the collision (this may be an important assumption). We want to solve for the ball's speed after

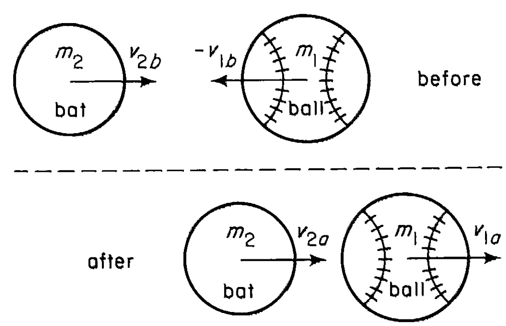

Fig. 1. In a collision between a ball (on the right and moving toward the left) and a bat (on the left and moving toward the right) momentum must be conserved. The subscript 1 is for the ball, 2 is for the bat. The subscript b is for before the bat-ball collision (the top diagram), and a is for after the collision (the bottom diagram)

its collision with the bat, called the *batted-ball speed*, but first we should eliminate the bat's speed after the collision, because it is not easily measured.

The coefficient of restitution, the bounciness of the ball, is defined as the relative speed after the collision divided by the relative speed before the collision. That is

$$e = -\frac{v_{1a} - v_{2a}}{v_{1b} - v_{2b}}. \tag{2}$$

We can solve (2) for v_{2a}, plug this into the conservation of momentum equation (1), and solve for the ball's speed after its collision with the bat

$$v_{1a} = \frac{(w_1 - ew_2)v_{1b} + (w_2 + ew_2)v_{2b}}{w_1 + w_2}. \tag{3}$$

Kirkpatrick (1963) assumed that the optimal bat weight would be the one that "requires the least energy input to impart a given velocity to the ball." This definition in conjunction with (3) yields

$$\left[\frac{w_2}{w_1}\right]_{\text{optimal}} = \frac{v_{1a} - v_{1b}}{v_{1a} + ev_{1b}}. \tag{4}$$

If we now make some reasonable assumptions like
$w_1 = 5.125$ oz, the weight of the ball.[1]
$e = 0.55$, the coefficient of restitution of a baseball.
$v_{1b} = -80$ mph, a typical pitch speed.
$v_{1a} = 110$ mph, the ball speed needed for a home run.

Then we can solve (4) to find that the *optimal bat weight* is 15 ounces!

Brancazio (1987) has written an excellent theoretical analysis of bat-ball collisions. He considered not only the bat's translation, but also its angular rotation about two axes. He found that the ball's speed after the collision with the bat depends on:

(1) the energy imparted by the body and arms;
(2) the energy imparted by the wrists;
(3) the speed of the pitch;
(4) the point of collision of the ball respect to
(a) the center of percussion,
(b) the center of mass,
(c) the end of the bat,
(d) the maximum energy transfer point; and also
(5) the weight of the bat.

However, generalizing over all these dependencies he also concluded that the *optimal bat weight* is about 15 ounces.

These conclusions cannot help professional baseball players, who must use solid wood bats, because a 15 ounce solid wood bat would only be 15 inches long! But this conclusion does help explain why people choke up on the bat; choking up makes the bat effectively shorter, moves the center of mass closer to the hands thereby reducing the moment of inertia, and in essence makes the bat act like a lighter bat. This conclusion could also help explain the great popularity of aluminum bats. The manufacturers can make them lighter while maintaining the same length and width. It might also explain why so many professional players are "corking" their bats (Gutman 1988; Kaat 1988).

However, both of these studies were limited by their explicit assumptions. Kirkpatrick assumed that the optimal bat was the one that required the least kinetic energy. And Brancazio's calculations for the optimal bat weight explicitly assumed that the "batter generates a fixed quantity of energy in a swing," independent of the bat weight. In this paper we will extend these calculations by allowing the amount of energy imparted to the bat by the batter to depend on bat weight.

Physiologists have long known that muscle speed decreases with increasing load (Fenn and Marsh 1935; Hill 1938; Jewell and Wilke 1960; Wilke 1950). This is why bicycles have gears: so the rider can keep muscle speed in its optimal range while bicycle speed varies greatly. We measured many batters swinging bats of various weights. We plotted the data of bat speed and bat weight, and used this to help calculate the best bat weight for each batter.

The Bat Chooser™ Instrument

Our instrument for measuring bat speed, the Bat Chooser,[2] had two vertical light beams each with an associated light detector (similar to an elevator door electric eye). The subjects swung a bat so that the center of mass of the bat passed through the light beams. The computer recorded the time between interruptions of the light beams. Knowing the distance between the light beams and the time required for the bat to travel that distance, the computer calculated the speed of the bat's center of mass for each swing.

We told the batters to swing each bat as fast as they could while still maintaining control. We told the professionals "Pretend you are trying to hit a Nolan Ryan fastball."

In our experiments each adult subject swung six bats through the light beams. The bats ran the gamut from super light to super heavy; yet they had similar lengths and weight distributions. In our developmental experiments we tried about three dozen bats. We used aluminum bats, wood bats, plastic bats, infield fungo bats, outfield fungo bats, bats with holes in them, bats

[1] Baseball is not a metric sport. So we have not translated our units into SI Units; we have left them in the common units that baseball players and spectators find familiar. One ounce (oz) is 0.0283 kg and one mile per hour (mph) is 0.447 m/s

[2] Bat Chooser is a trademark of Bahill Intelligent Computer Systems, a patent is pending

Table 1. Characteristics of the six bats

Name	Weight (oz)	Length (inches)	Distance from the end of the handle to center of mass (inches)	Composition
D	49.0	35.0	22.5	Aluminum bat filled with water
C	42.8	34.5	24.7	Wood bat, drilled and filled with lead
A	33.0	35.5	23.6	Regular wood bat
B	30.6	34.5	23.3	Regular wood bat
E	25.1	36.0	23.6	Wood fungo bat
F	17.9	35.7	21.7	A wooden bat handle mounted on a threaded steel lamp pipe with a 6 oz weight attached to the end

Table 2. Characteristics of the four boy's bats

Name	Weight (oz)	Length (inches)	Distance to center of mass (inches)	Composition
A	40.2	29.9	17.8	Wood bat with iron collar
C	25.1	28.0	17.3	Wood bat
D	21.1	28.8	17.0	Aluminum bat
B	5.2	31.3	17.6	Plastic bat

with lead in them, major league bats, college bats, softball bats, Little League bats, brand new bats and bats over 40 years old. In many experiments we used the six bats described in Table 1. These bats were about 35 inches long, with the center of mass about 23 inches from the end of the handle.

For Little League players we changed to a different set of bats; they had to be lighter and fewer in number. For our final experiments we used the set described in Table 2. Even with this set we still saw signs of fatigue in half our subjects.

In a 20 min interval of time, each subject swung each bat through the instrument five times. The order of presentation was randomized. The selected bat was announced by a DECtalk® speech synthesizer as follows: "Please swing bat Hank Aaron, that is, bat A." We recorded the bat weight and the linear velocity of the center of mass for each swing.

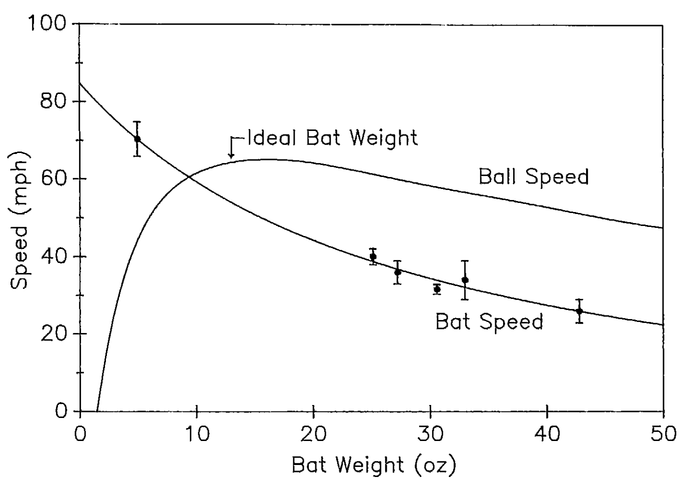

Fig. 2. Bat speed and calculated ball speed after the collision both as functions of bat weight for a 40 mph pitch to Alex, a ten year old Little League player. The dots represent the average of the five swings of each bat; the vertical bars on each dot represent the standard deviations. These data were collected with a different set of bats than that described in Table 2

The Force-Velocity Relationship of Physiology

When bat speeds measured with this instrument were plotted as a function of bat weight we got the typical muscle force-velocity relationship shown in Fig. 2.

The ball speed curve and the term ideal bat weight shown in this figure will be discussed in a later section. This force-velocity relationship shows that the kinetic energy ($1/2\ mv^2$) put into a swing was zero when the bat weight was zero, and also when the bat was so heavy that the speed was reduced to zero. The bat weight that allows the batter to put the most energy into the swing, the *maximum-kinetic-energy bat weight*,

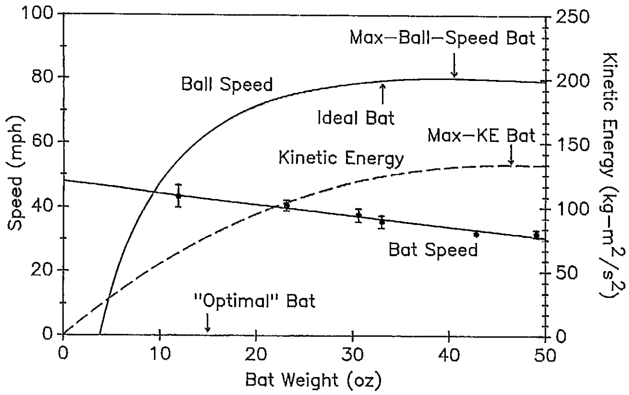

Fig. 3. Bat speed, kinetic energy given to the bat, and calculated ball speed after the collision, all as functions of bat weight for an 90 mph pitch for a member of the San Francisco Giants baseball team. Data for other professional baseball players were similar. These data were collected with a different set of bats than that described in Table 1

occurred somewhere in between. This led to the suggestion that the batter might choose a bat that would allow maximum kinetic energy to be put into the swing. Figure 3 shows the kinetic energy (dashed line) as a function of bat weight for a member of the San Francisco Giants: this batter could impart the maximum energy to a bat weighing 46.5 oz.

However, this maximum-kinetic-energy bat weight does not tell us the bat weight that will make the ball go the fastest. To calculate this weight we must couple the muscle force-velocity relationship to the equations for conservation of momentum. Then we can solve the resulting equations to find the bat weight that would allow a batter to produce the greatest batted-ball speed. This would, of course, make a potential home run go the farthest, and give a ground ball the maximum likelihood of getting through the infield. We call this weight the *maximum-batted-ball-speed bat weight.*

Coupling Physiology to Physics

Next we coupled the physiology to the equations of physics. First, we fit three different, physiologically realistic (Agarwal and Gottlieb 1984), equations to the data for the 30 swings. We fit a straight line $y = Ax + B$, a hyperbola $(x+A)(y+B) = C$, and an exponential $y = Ae^{-Bx} + C$. Then we chose the equation that gave the best fit; for the data of Fig. 3, the best fit was: bat speed (in mph) $= -0.34$ bat weight (in oz) $+48$, or

$$v_{2b} = -0.34 w_2 + 48 . \tag{5}$$

Next we substituted this relationship into (3) yielding

$$v_{1a} = \frac{(w_1 - e w_2) v_{1b} + (w_2 + e w_2)(A w_2 + B)}{w_1 + w_2} . \tag{6}$$

Then we took the derivative with respect to the bat weight, set this equal to zero, and solved for the maximum-batted-ball-speed bat weight.

$$w_{2mbbs} = \frac{-w_1 A - \sqrt{w_1^2 A^2 - A w_1 (B - v_{1b})}}{A} . \tag{7}$$

For the data of Fig. 3 this was 40.5 oz.

The physics of bat-collision predicts an optimal bat weight of 15 oz. The physiology of the muscle force-velocity relationship reveals a maximum-kinetic-energy bat weight of 46.5 oz for the professional baseball player of Fig. 3. When we coupled (5), fit to the force-velocity data of Fig. 3, to the equation derived from the coefficient of restitution and the principle of conservation of momentum for bat-ball collisions (3), we were able to plot the ball speed after the collision (called the batted-ball speed) as a function of bat weight, also shown in Fig. 3. This curve shows that the maximum-batted-ball-speed bat weight for this subject was 40.5 oz, which is heavier than that used by most batters. However, this batted-ball speed curve is almost flat between 34 and 49 oz. There is only a 1.3% difference in the batted-ball speed between a 40.5 oz bat, and the 32 oz bat normally used by this player. Evidently the greater control permitted by the 32 oz bat outweighs the 1.3% increase in speed that could be achieved with the 40.5 oz bat.

Ideal Bat Weight™

The maximum-batted-ball-speed bat weight is not the best bat weight for any player. A lighter bat will give a player better control and more accuracy. So obviously a trade-off must be made between maximum batted-ball speed and controllability. Because the batted-ball speed curve of Fig. 3 is so flat around the point of the maximum-batted-ball-speed bat weight, we believe there is little advantage to using a bat as heavy as the maximum-batted-ball-speed bat weight. Therefore, we defined the *ideal bat weight*[3] to be the weight where the ball speed curve drops 1% below the maximum speed.[4] Using this criterion, the ideal bat weight for this subject is 33 oz. We believe this gives a reasonable trade-off between distance and accuracy. Of course this is subjective and each player might want to weigh the two factors differently. But at least this gives a quantitative basis for comparison. The player of Fig. 3 was typical of the San Francisco Giants that we measured, as

[3] Ideal bat weight is a trademark of Bahill Intelligent Computer Systems

[4] A sensitivity analysis has shown that this 1% figure is the most important parameter in the model. In future experiments we will derive a curve for accuracy versus bat weight and use this data instead

Table 3. Summary data for the 28 San Francisco Giants

	Average	Range
Maximum kinetic energy (joules)	270	133–408
Maximum batted-ball speed (mph)[a]	99	80–122
Ideal bat weight (oz)	31.7	26.25–37.00
Actual bat weight (oz)	32.3	31–34

[a] A batted-ball speed of 110 mph is needed for a home run

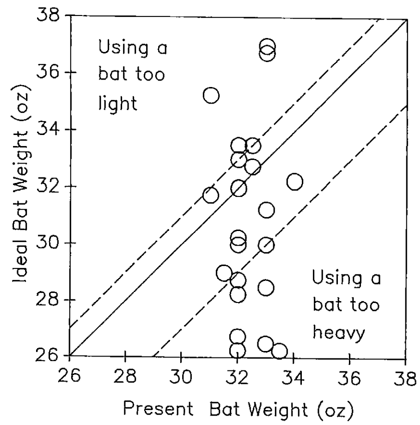

Fig. 4. Ideal bat weight versus actual bat weight for the San Francisco Giants. Most of them are now using bats in their recommended range

shown in Table 3, except that his swings were slower but more consistent than most. He is a control hitter.

The ideal bat weight varies from person to person. Table 4 shows the mean and standard deviation of the ideal bat weight for batters in various organized leagues. These calculations were made with the pitch speed each player was most likely to encounter, i.e., 40 mph for Little League and 20 mph for university professors playing slow pitch softball.[5] Ideal bat weight is specific for each individual, but it is not correlated with height, weight, age or any combination of these factors, nor is it correlated with any other obvious physical factors.

To further emphasize the specificity of the ideal bat weight calculations, we must display individual statistics, not averages and standard deviations. So in Fig. 4 we show the ideal bat weight as a function of the weight of the actual bat used by the players before our experiments.

This figure shows that most of the players on the San Francisco Giants baseball team are using bats in the correct range; the dashed lines in this figure (derived from data and calculations not shown in this paper) represent our recommendations to management. We recommended that batters above the upper dashed line switch to heavier bats, and that batters below the lower dashed line switch to lighter bats.

[5] The coefficient of restitution of a softball is small than that of a baseball, but this did not effect our calculations, because the ideal bat weight is independent of the value of the coefficient of restitution

Not only is the ideal bat weight specific for each player, but it also depends on whether the player is swinging right or left handed. We measured two switch hitters: one's ideal bats weights were one ounce different and the other's were 5 ounces different. Switch hitters were so different right and left handed that we treated them as different players.

Extrapolating from (7) shows that the ideal bat weight also depends on pitch speed. Figure 5 shows this dependence of ideal bat weight on pitch speed for the ball player of Fig. 3. This figure also shows the resulting batted-ball speed after a collision with a bat of the ideal weight. Such curves were typical of all our subjects.

This figure shows that the ideal bat weight increases with increasing pitch speed. Which means that even if they could swing 33 oz bats, Little Leaguers should use lighter bats, because the pitch speeds are lower. However, when this knowledge is used to identify the ideal bat weight for a particular individual, the results may seem counter-intuitive. When the opposing pitcher is a real fireballer, the coach often says "Choke up (i.e. get a lighter bat), so you can get around on it." In such situations we think the coach is changing the subjective weighting of bat control versus distance. He is saying drop your criterion to 2 or 3% below maximum-batted-ball-speed bat weight so you

Table 4. Measured ideal bat weight

Team	Mean ideal bat weight (oz)	Standard deviation	Pitch speed	Number of subjects
San Francisco Giants	31.7	3.8	90	28
University baseball	28.3	2.8	80	11
University softball	27.8	3.7	60	12
Little League	20.1	3.4	40	11
Slow pitch softball	19.4	1.0	20	4

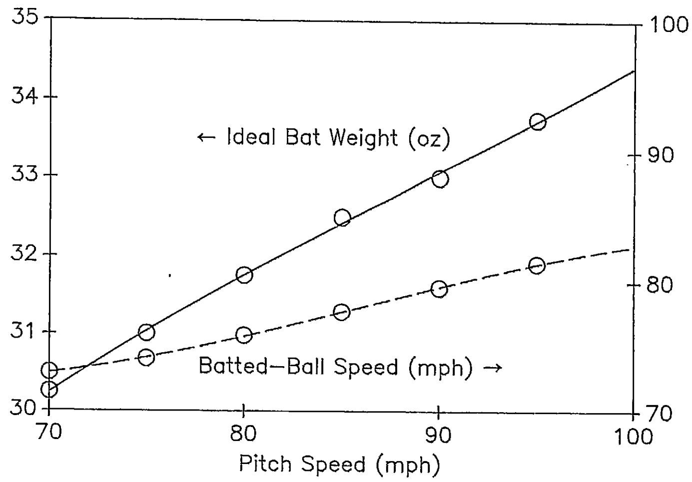

Fig. 5. Ideal bat weight and batted-ball speed both as a function of pitch speed for the professional baseball player of Fig. 3

can get better bat control. After all, the batted-ball speed depends on both the pitch speed and the bat weight. So the batter can afford to choke-up with a fast pitcher, knowing that the ball will go just as fast as not choking-up with a slower pitcher.

One more observation that came from our studies is that proficient batters have very consistent swings; they are machine like. They train this machine-like precision. Compare the height of the data crosses (the standard deviation) of the swings of the typical Little League player of Fig. 2, who had been playing ball for five years, with the height of the crosses for the swings of the professional player who had been playing ball for 24 years (Fig. 3). Consistency is important and the professional player shows this consistency.

Generalizations and Limitations

It is not surprising that in a game that is more than 100 years old, with players being paid hundreds of thousands of dollars per year, that professional athletes, without the benefit of scientists and engineers, have found that their best bat weights are between 30 and 34 oz. However, it is interesting to note that, given the relative newness of the aluminum bat, and the fact that they are used by amateurs, that Little League and slow pitch softball players cannot yet get bats that are light enough for them. The lightest Little League approved bat that we have seen is 21 oz. The lightest legal softball bat that we have seen is 27 oz. (However, these numbers are decreasing at the amazingly high rate of about 2 ounces per year.)

For the Little Leaguer of Fig. 2 the batted-ball speed varies greatly with bat weight. This means it is very important for him to have the right weight bat. However, for most professional baseball players, once the bat is in the correct range, the batted-ball speed varies little with bat weight as shown in Fig. 3. That player could use any bat in the range 33 to 40 ounces and there would be less than a one percent change in batted-ball-speed. This fact is not in the literature and it could not be determined by experimentation. For example, imagine an experiment where a pitcher alternately throws 20 white balls and 20 yellow balls to a batter who alternately hits with a 32 or a 34 ounce bat. Imagine then going into the outfield and looking at the distribution of the balls. You would not see the yellow balls or the white balls consistently farther out. Variability in the pitch and the location of the contact point between the bat and the ball would obscure any differences. However, in our instrument we can accurately measure bat speed and calculate the resulting batted-ball speed. Our calculations show that this curve is flat. This knowledge should help batters eliminate futile experimentation altering bat weights, trying to get higher batted-ball speeds. As long as the player uses a bat in the flat part of his curve there will be less than a one percent variation in batted-ball speed caused by varying bat weights. What does a one percent decrease in batted-ball speed mean? A ball that would normally travel 333 feet would only travel 330 feet. This does not seem important.

In our studies we measured bat speed as a function of bat weight. Next, we coupled these measurements to the equations of physics and physiology to determine the ideal bat weight for each individual batter. We can say nothing about the "feel" of a bat; this is a psychological variable that we cannot measure. We have no means of assessing the accuracy of the swing. Throughout our analysis we assume that it is easier to control a lighter bat than a heavier one. We are not concerned with the availability of bats. Our recommendations are independent of what equipment is actually available. We try hard to make sure that our solutions to tomorrow's problems are not stated in terms of yesterday's hardware.

In this study we measured the linear velocity of the center of mass of the bats. It is obvious that in addition to this translation the bat also rotates about two different axes. However, our results derived from only linear velocity agree for most details with those Brancazio derived using angular velocity. The exception is that for a rotating bat, the place where the ball hits the bat becomes important. If the ball hits the bat at its center of mass the results of the linear approximation are the same as those derived considering the rotations. However, if the ball hits the bat six inches closer to its end then the ideal bat weight would increase two ounces for the Little Leaguer of Fig. 2, and three ounces for the major leaguer of Fig. 3. All in all, we think our approximation of linear velocity only is reasonable.

We also neglected the effect of air resistance on pitch speed. We calculated the ideal bat weights of the major league players based on a pitch speed of 90 mph. If the ball was going 90 mph when it hit the bat it would indeed be a fast pitch, because the ball loses about 10% of its speed on its way to the plate. If we decreased our 90 mph figure by 10%, the ideal bat weights would decrease by an ounce. This change partially cancels the correction mentioned in the above paragraph.

Our data have low variability for physiological data: for the data of Fig. 3 the standard deviations are about $\pm 5\%$. However, the repeatability of our experiments is not as good. On any given day the data are repeatable. But tests run 1, 2 or 12 months apart differ by as much as 20% in bat speed for any given bat. However, in spite of these large differences in bat speed, the calculated ideal bat weight varies by only an ounce or two. We are still looking for the sources causing the lack of repeatability. We think the most likely causes are warmup condition, adrenlin, positioning in the instrument, and fatigue.

Our experiments were done indoors; some of the ball players thought things would be different out on the field swinging against a real pitcher. So we took the equipment out to the ball field. Right after an intra-squad game, we measured the bat speeds of four members of the University of Arizona baseball team while they hit the baseball. For each player these data fell within the range of his data collected in the laboratory six months earlier and one week later.

Adequacy of Muscle Models

We had reservations about using the Hill equation for our muscle model. First of all, most data for muscle force-velocity relationships come from single muscles or fibers, and our data are for multi-joint movements. Second, most data for muscle force-velocity relationships come from isolated frog or rat muscle at 10°C, and our data are for whole intact human beings at 38°C. Third, traditional force-velocity curves are derived from experiments where the muscle lifts weights against the force of gravity. Only the weight is important; i.e. inertia viscosity, and elasticity are ignored. For example, in a typical muscle model the tension in the tendon is

$$T = F + Kx - B\dot{x} = M\ddot{x} + Mg.$$

And the active state tension of the muscle is

$$F = -Kx + B\dot{x} + M\ddot{x} + Mg.$$

For typical physiological experiments, e.g., (Jewell and Wilke 1960) the inertial component, $\ddot{x} \approx 100\ \mathrm{mm/s^2}$, is only 1% of the gravitational component, $g = 9800\ \mathrm{mm/s^2}$. In experiments where the inertial term was larger, its effects were often subtracted off before the force-velocity data were plotted (Wilke 1950). Making reasonable approximations for B and K shows that $B\dot{x}$ and Kx are only 1 or 2% of $M\ddot{x}$. Therefore, for typical physiological experiments the weight of the load is much more important than the inertial, viscous, and elastic terms. However, gravity has no effect in swinging a baseball bat. The swing of the baseball bat is horizontal; the batter does not fight gravity at all.

As previously noted we fit three different equations to our force-velocity data: straight lines, hyperbolas, and exponentials. We found no interesting differences between the hyperbolic and exponential fits. We also fit the data with a more complicated model. We used a model that considered the force-velocity relationship, the length-tension diagram, the parallel elasticity, the series elasticity, the roles of agonist and antagonist muscles, etc.; i.e. we used a modified version of the 18 parameter linear homeomorphic model (Bahill et al. 1980). This model produced data that were indistinguishable from the hyperbolic and exponential fits. Therefore, from now on we will treat the exponential, the complicated model and the hyperbolic as one class and call them hyperbolic.

We found two types of force-velocity relationships: those that were fit best with a straight line of low slope (like Fig. 3), and those where the straight line fits had high slopes, but more importantly the hyperbolic fits were much better (in a mean squared error sense) (like Fig. 2).

Figure 2 shows a highly sloped force-velocity relationship exhibited by a quick Little Leaguer. The data are best fit with

$$(w_{\mathrm{bat}} + 28.0) \times (\mathrm{speed} + 12.8) = 2728.$$

The data of his brother, also a Little Leaguer, collected on the same day are best fit with a straight line.

This division into two groups also holds for members of the San Francisco Giants baseball team, as shown in Fig. 6. The top figure is for a quick player: the hyperbolic fit (solid line) is 35% better (in a mean squared error sense) than the straight line fit (dotted line). The bottom figure is for a slugger, his data are fit best with a straight line.

Most Little Leaguers were fit best with hyperbolas: half of our college players were fit best with hyperbolas: one-fourth of our major leaguers were fit best with hyperbolas. For 22 of the 28 San Francisco Giants the straight line and hyperbolic fits were just as good (within 5%). For the other 6 the hyperbolic fits were much better. For these six the percentage superiority of the hyperbolic fits were: 11%, 18%, 20%, 23%, 27%, and 35%.

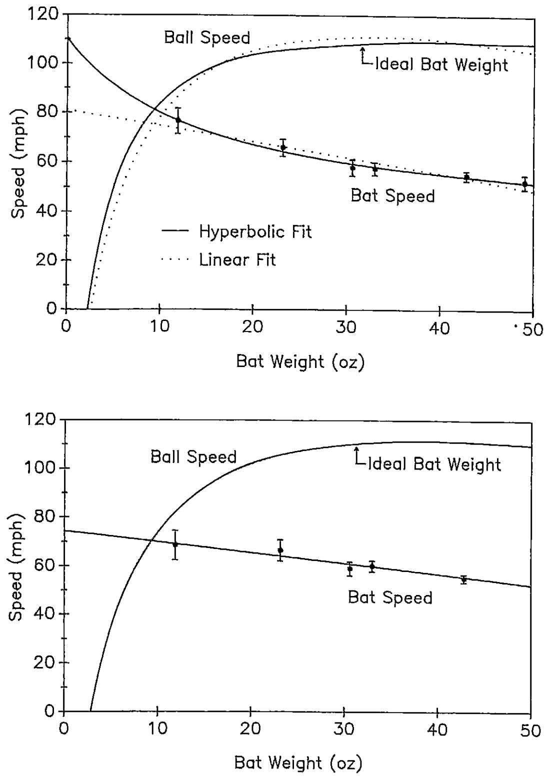

Fig. 6. Bat speed, and batted-ball speed as functions of bat weight for an 90 mph pitch for two different members of the San Francisco Giants baseball team. The player of the top graph was a quick, singles hitter. The player of the bottom graph was a slugger. These data were collected with a different set of bats than that described in Table 1

We tried to correlate the slope of the straight line fit with height, weight, body density, arm circumference, present bat weight, running speed, etc., but had no success. However, we noted that the subjects who had large slopes were described by their coaches as being "quick." Quickness is not the same as running speed, but it is related. Quickness is easy to identify but hard to define. Coaches easily identified their quick players, but when asked to explain why they called these players quick, they waffled. Their uneasy verbalizations include phrases like they react quickly, they move fast, they steal many bases, they get into position to field the ball quickly, they swing the bat fast, and they beat out bunts. But all these phrases describe resulting behavior, not physiological characteristics. So we decided to measure eye-hand reaction time and try to correlate it with the bat swing data. Eye-hand reaction time was measured by: *(1)* Holding a meter stick in front of a subject. Instructing him to place his opened index finger and thumb at the 50 cm mark and watch the fingers of the experimenter, who is holding the end of the meter stick. *(2)* When the experimenter opens his fingers and the meter stick begins to fall the subject should close his fingers. *(3)* The place on the meter stick where he catches it indicates eye-hand reaction time ($d = 1/2\ at^2$). *(4)* Each subject was given two warm up trials, then we collected data for ten trials. Then we selected the median value of these ten trials.

The eye hand reaction time for the quick boy of Fig. 2 was 143 ms, for his nonquick brother it was 256 ms. We collected eye-hand reaction times for 21 of the San Francisco Giants ($\bar{x} = 158$ ms, $\sigma = 24$) and compared it with the percentage superiority of the hyperbolic fit. We applied a linear regression analysis to this data and found

$$\text{reaction time (ms)} = -1.04(\text{percentage superiority}) + 164\,.$$

The correlation coefficient, r, was -0.4. Now it is obvious that this is not a large slope or a huge correlation, but in our data base the only correlation that was bigger was

$$\text{slope of straight line fit} = 0.009(\text{percentage superiority}) + 0.58$$

that had $r = 0.55$. The slope of the straight line is not a physiological parameter. So we conclude that the physiologic parameter that best differentiates between players whose data can be best fit with a straight line and those who require a hyperbola is the eye-hand reaction time.

For our nonquick subjects the weight of the bat seemed to have little effect on how they swung it. They swung all bats with about the same speed, and their data were fit best with a straight line, as shown in Fig. 3. For our quick subjects the weight of the bat was a limiting factor. Speed depended on weight. The curves had steep slopes and needed hyperbolas to fit the data, as shown in Fig. 2. We hypothesize that the quick people change their control strategies when given a different bat. Whereas nonquick people do not change their strategies, they swing all bats the same.

Our experiments only covered a small part of the possible range of bat weights; we restricted our data to the physiological range. For some experiments we tried using heavier bats. But, as expected, the fits were not as good. We noticed that the batters changed their strategies between swings. For the super heavy bats they swung more with their feet and body and less with their arms. Therefore, we discontinued the use of super heavy bats because we did not want our data to contain swings performed with different body strategies. Most adults could handle bats up to 50 oz, and most kids could handle bats up to 40 oz.

This discussion about muscle models is important for an understanding of the human neuromuscular system. However, pragmatically it is insignificant, because all models predicted about the same ideal bat

Table 5. Specific values for figures of this paper

Team	Figure no.	Maximum kinetic energy bat weight (oz)	Maximum batted ball speed bat weight (oz)	Ideal bat weight (oz)	Actual bat weight (oz)
Little League	2	26	16	15	21
San Francisco Giants	3	46	40	33	32
San Francisco Giants	6 top	42	32	27	33
San Francisco Giants	6 bottom	58	39	32	33

weight. For example, for the data of Fig. 3, the ideal bat weight predicted by the linear, hyperbolic, and exponential fits are respectively 33, 33.75, and 33.75 oz. Even for the data of Fig. 2 the three models yield similar results of 14.75, 12.75, and 12.75 oz.

Discussion

The physics of bat-ball collisions (specifically the equations for conservation of momentum and the coefficient of resititution) predicted an *optimal bat weight* of 15 oz. The physiology of the muscle force-velocity relationship showed that the professional baseball player of Fig. 3 could put the most energy into a swing with a 46 oz bat, i.e. his *maximum-kinetic-energy bat weight* was 46 oz. Coupling physics to physiology showed his *maximum-batted-ball-speed bat weight* to be 40 oz. Finally trade-offs between maximum ball speed and controllability showed that his *ideal bat weight* was 33 oz, which is close to his actual bat weight of 32 oz. These experiments explain why most adult batters use bats in the 28 to 34 oz range, they explain the variability in human choice of bat weight, and they suggest that there is an ideal bat weight for each person. However, they leave unanswered questions about quickness, changes in control strategy, and the need for hyperbolic curves to fit muscle force-velocity data.

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Prof. A. Terry Bahill
Systems and Industrial Engineering
University of Arizona
Tucson, AZ 85721
USA