16

Mechanics of Baseball Pitching and Batting

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CONTENTS

This chapter discusses the pitch, the bat–ball collision, and the swing of the bat. Section 16.1, based on Bahill and Baldwin [1], describes the pitch in terms of the forces on the ball and the ball's movement. Section 16.2, based on Bahill [2] and Bahill and Baldwin [3], discusses bat–ball collisions in terms of the sweet spot of the bat and the coefficient of restitution (CoR). Section 16.3 based on Bahill and Baldwin [3], presents a model for bat–ball collisions, a new performance criterion, and the resulting vertical sweet spot of the bat. Section 16.4, based on Bahill [2] and Bahill and Karnavas [4], presents experimental data describing the swing of a bat and suggests ways of choosing the best bat for individual batters.

This chapter is about the mechanics of baseball. To understand the whole baseball enterprise, read Bahill et al. [5]. They populate a Zachman framework with nearly 100 models of nearly all aspects of baseball.

16.1 Pitch

Batters say that the ball hops, drops, curves, breaks, rises, sails, or tails away. The pitcher might tell you that he or she throws a fastball, screwball, curveball, drop curve, flat curve, slider, changeup, split-fingered fastball, splitter, forkball, sinker, cutter, two-seam fastball, or four-seam fastball. This sounds like a lot of variation. However, no matter how the pitcher grips or throws the ball, once it is in the air its motion depends only on gravity, its velocity, and its spin.* In engineering notation, these pitch characteristics are described respectively by a ''linear velocity vector'' and an ''angular velocity vector,'' each with magnitude and direction. The magnitude of the linear velocity vector is called ''pitch speed'' and the magnitude of the angular velocity vector is called the ''spin rate.'' These vectors produce a force acting on the ball that causes a deflection of the ball's trajectory.

In 1671, Isaac Newton [6] noted that spinning tennis balls experienced a lateral deflection mutually perpendicular to the direction of flight and of spin. Later in 1742, Benjamin Robins [7] bent the barrel of a musket to produce spinning musket balls and also noted that the spinning balls experienced a lateral deflection perpendicular to the direction of flight and to the direction of spin. In 1853, Gustav Magnus (see Refs. [8 and 9]) studied spinning shells fired from rifled artillery pieces and found that the range depended on crosswinds. A crosswind from the right lifted the shell and gave it a longer range: a crosswind from the left made it drop short. Kutta and Joukowski studied cylinders spinning in an airflow. They were the first to model this force with an equation, in 1906. Although these four experiments sound quite different (and they did not know about each other's

* This statement is true even for the knuckleball, because it is the shifting position of the seams during its slow spin en route to the plate that gives the ball its erratic behavior. Equations 16.1 through 16.7 give specific details about the forces acting on the ball.

papers), they were all investigating the same underlying force. This force, commonly called the Magnus force, operates when a spinning object (like a baseball) moves through a fluid (like air) which results in it being pushed sideways. Two models explain the basis of this Magnus force: one is based on conservation of momentum and the other is based on Bernoulli's principle [10–12]. We will now apply the right-hand rules to the linear velocity vector and the angular velocity vector in order to describe the direction of the spin-induced deflection of the pitch.

16.1.1 Right-Hand Rules and the Cross Product

In vector analysis, the right-hand rules specify the orientation of the cross product of two vectors. Figure 16.1a shows that the cross (or vector) product, written as $\mathbf{u} \times \mathbf{v}$, of nonparallel vectors $\mathbf{u}$ and $\mathbf{v}$ is perpendicular to the plane of $\mathbf{u}$ and $\mathbf{v}$: the symbol $\times$ represents the cross product. The angular right-hand rule, illustrated in Figure 16.1b, is used to specify the orientation of a cross product $\mathbf{u} \times \mathbf{v}$. If the fingers of the right hand are curled in the direction from $\mathbf{u}$ to $\mathbf{v}$, the thumb will point in the direction of the vector $\mathbf{u} \times \mathbf{v}$. The coordinate right-hand rule is illustrated in Figure 16.1c. The index finger, middle finger, and thumb point in the directions of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} \times \mathbf{v}$, respectively, in this local coordinate system. The vectors of Figure 16.1d represent the angular velocity vector (spin), the linear velocity vector (direction), and the spin-induced deflection force of a spinning pitch.

16.1.2 Right-Hand Rules Applied to a Spinning Ball

The spin axis of the pitch can be found by using the angular right-hand rule. As shown in Figure 16.2, if you curl the fingers of your right hand in the direction of spin, your extended thumb will point in the direction of the spin axis.

The direction of the spin-induced deflection force can be described using the coordinate right-hand rule. Point the thumb of your right hand in the

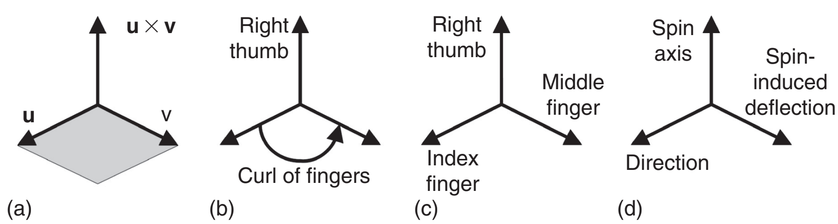

FIGURE 16.1
(a) The vector (or cross) product of vectors $\mathbf{u}$ and $\mathbf{v}$ is perpendicular to the plane of $\mathbf{u}$ and $\mathbf{v}$. (b) The angular right-hand rule: If the fingers of the right hand are curled in the direction from $\mathbf{u}$ to $\mathbf{v}$, the thumb will point in the direction of the vector $\mathbf{u} \times \mathbf{v}$, which is pronounced *yōō cross vē*. (c) The coordinate right-hand rule: The index finger, the middle finger, and the thumb point in the directions of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} \times \mathbf{v}$, respectively. (d) For a baseball, the cross product of the spin axis and the direction of motion gives the direction of the spin-induced deflection. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

FIGURE 16.2
The angular right-hand rule: For a rotating object, if the fingers are curled in the direction of rotation, the thumb points in the direction of the spin axis. (Photograph courtesy of Zach Bahill. From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

direction of the spin axis (as determined from the angular right-hand rule), and point your index finger in the direction of forward motion of the pitch (Figure 16.3). Bend your middle finger so that it is perpendicular to your index finger. Your middle finger will be pointing in the direction of the spin-induced deflection (of course, the ball also drops due to gravity). The spin-induced deflection force will be in a direction represented by the cross product of the angular and the linear velocity vectors of the ball: angular velocity $\times$ linear velocity = spin-induced deflection force. Or mnemonically, Spin axis $\times$ Direction = Spin-induced deflection (SaD Sid). This acronym only gives the direction of deflection. The equation yielding the magnitude of the spin-induced deflection force is more complicated and is discussed in Section 16.1.4.

16.1.3 Deflection of Specific Kinds of Pitches

Figures 16.4 and 16.5 show the directions of spin (circular arrows) and spin axes* (straight arrows) of some common pitches from the perspective of the pitcher (Figure 16.4 represents a right-hander's view and Figure 16.5 a

* These could be labeled spin vectors, because they suggest both magnitude and direction.

FIGURE 16.3
The coordinate right-hand rule: For a baseball, if the thumb points in the direction of the spin axis and the index finger points in the direction of forward motion of the pitch, then the middle finger will point in the direction of the spin-induced deflection. (Photograph courtesy of Zach Bahill. From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

left-hander's view). We will now consider the direction of deflection of each of these pitches.

Figure 16.4 illustrates the fastball, curveball, and slider, distinguished by the direction of the spin axis. When a layperson throws a ball, the fingers are the last part of the hand to touch the ball. If the ball is thrown with an overhand motion, the fingers touch the ball on the bottom and thus impart backspin to the ball. Most pitchers throw the fastball with a three-quarter arm delivery, which means the arm does not come straight over the top, but rather it is in between over the top and sidearm. This delivery rotates the spin axis from the horizontal as shown in Figure 16.4. The curveball is also thrown with a three-quarter arm delivery, but this time the pitcher rolls his or her wrist and causes the fingers to sweep in front of the ball. This produces a spin axis as shown for the curveball of Figure 16.4. This pitch

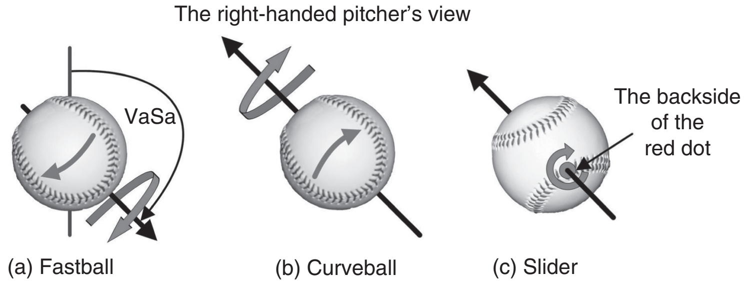

FIGURE 16.4 (See color insert following page xxx.)
The direction of spin (circular arrows) and the spin axes (straight arrows) of a three-quarter arm (a) fastball, (b) curveball, and (c) slider from the perspective of a right-handed pitcher, meaning the ball is moving into the page. VaSa is the angle between the vertical axis and the spin axis. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2005.)

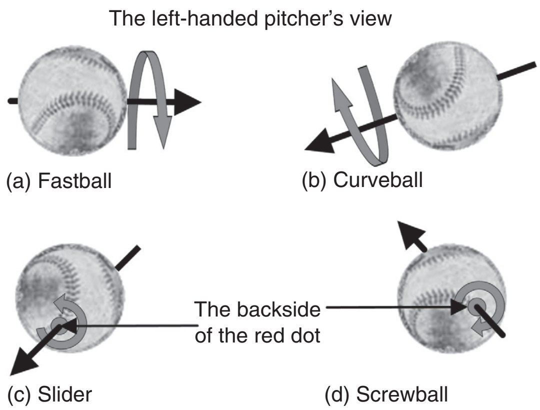

FIGURE 16.5 (See color insert following page xxx.)
The direction of spin (circular arrows) and the spin axes (straight arrows) of an overhand (a) fastball, (b) curveball, (c) slider, and (d) screwball from the perspective of a left-handed pitcher, meaning the ball is moving into the page. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

will curve at an angle from upper right to lower left as seen by a right-handed pitcher. Thus, the ball curves diagonally. The advantage of the drop in a pitch is that the sweet area of the bat is about 2 in. long (5 cm) [2] but only one-third of an inch (8 mm) high [3,13]. Thus, when the bat is swung in a horizontal plane, a vertical drop is more effective than a horizontal curve at taking the ball away from the bat's sweet area.

The overhand fastball shown in Figure 16.5 has a predominate backspin, which gives it lift, thereby decreasing its fall due to gravity. But when the fastball is thrown with a three-quarter arm delivery (as in Figure 16.4), the lift is reduced, but it introduces lateral deflection (to the right for a right-handed pitcher). A sidearm fastball (from a lefty or a righty) tends to have some topspin, because the fingers put pressure on the top half of the ball during the pitcher's release. The resulting deflection augments the effects of gravity and the pitch ''sinks.''

The slider is thrown somewhat like a football. Unlike the fastball and curveball, the spin axis of the slider is not perpendicular to the direction of forward motion (although the direction of deflection is still perpendicular to the cross product of the spin axis and the direction of motion). As the angle between the spin axis and the direction of motion decreases, the magnitude of deflection decreases, but the direction of deflection remains the same. If the spin axis is coincident with the direction of motion, as for the backup slider, the ball spins like a bullet and undergoes no deflection. Therefore, a right-handed pitcher usually throws the slider so that he or she sees the axis of rotation pointed up and to the left. This causes the ball to drop and curve from the right to the left. Rotation about this axis allows some batters to see a red dot at the spin axis on the top right side of the ball (see Figure 16.6). Baldwin et al. [14] and Bahill et al. [15] show pictures of this spinning red dot. Seeing this red dot is important, because if the batter can see this red

FIGURE 16.6
The batter's view of a slider thrown by a right-handed pitcher: the ball is coming out of the page. The red dot reveals that the pitch is a slider. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

dot, then he or she will know the pitch is a slider and he or she can better predict its trajectory. We questioned 15 former major league hitters; 8 remembered seeing this dot, but 2 said it was black or dark gray rather than red. For the backup slider, the spin causes no horizontal deflection and the batter might see a red dot in the middle of the ball.

16.1.4 Forces Acting on a Ball in Flight

A ball in flight is influenced by three forces as shown in Figure 16.7: gravity pulling downward, air resistance or drag operating in the opposite direction of the ball's motion, and, if it is spinning, a force perpendicular to the direction of motion. The force of gravity is downward, $F_{\text{gravity}} = mg$, where m is the mass of the ball and g is the gravitation constant: its magnitude is the ball's weight. The magnitude of the force opposite to the direction of flight is

$$F_{\text{drag}} = 0.5\ \rho\pi r_{\text{ball}}^2 C_{\text{d}} v_{\text{ball}}^2 \qquad (16.1)$$

where
ρ is air mass density
v_{ball} is the ball speed
r_{ball} is the radius of the ball [10, p. 161]

Typical values for these parameters are given in Table 16.1. Of course SI units can be used in this equation, but if English units are to be used in Equations 16.1 through 16.7, then ρ is measured in lb-s^2/ft^4, v_{ball} is measured in feet per second (ft/s), r_{ball} is measured in feet (ft), F_{drag} is measured in

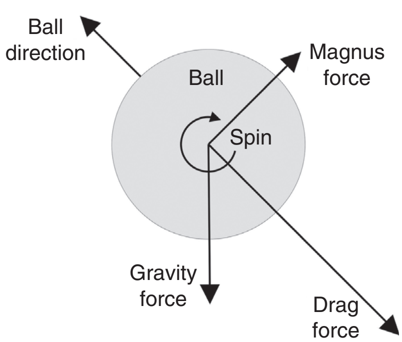

FIGURE 16.7
The forces acting on a spinning ball moving in a fluid. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2007.)

TABLE 16.1

Typical Baseball and Softball Parameters for Line Drives

	Major League Baseball	Little League	NCAA Softball
Ball	Baseball	Baseball	Softball
Ball weight (oz)	5.125	5.125	6.75
Ball weight, $F_{gravity}$ (lb)	0.32	0.32	0.42
Ball radius (in.)	1.45	1.45	1.9
Ball radius, r_{ball} (ft)	0.12	0.12	0.16
Pitch speed (mph)	85	50	65
Pitch speed, v_{ball} (ft/s)	125	73	95
Distance from front of rubber to tip of plate (ft)	60.5	46	43
Pitcher's release point: (distance from tip of plate, height) (ft)	(55.5, 6)	(42.5, 5)	(40.5, 2.5)
Bat–ball collision point: (distance from tip of plate, height) (ft)	(3, 3)	(3, 3)	(3, 3)
Bat type	Wooden C243	Aluminum	Aluminum
Typical bat weight (oz)	32	23	25
Maximum bat radius (in.)	1.375	1.125	1.125
Speed of sweet spot (mph)	60	45	50
Coefficient of restitution (CoR)	0.54	0.53	0.52
Backspin of batted ball (rps)	10–70	10–70	10–70
Backspin of batted ball, ω (rad/s)	63–440	63–440	63–440
Desired ground contact point from the plate (ft)	120–240	80–140	80–150
Air weight density (lb_m/ft^3)	0.075	0.075	0.075
Air mass density ρ ($lb\text{-}s^2/ft^4$)	0.0023	0.0023	0.0023

Note: Air density is inversely related to temperature, altitude, and humidity.

pounds (lb), and in later equations ω is measured in radians per second (rad/s). For the drag coefficient, $C_{d,}$ we use a value of 0.5. This drag coefficient is discussed in Section 16.1.7.

Table 16.2 shows typical parameters for major league pitches. We estimate that 90% of major league pitches fall into these ranges, except for a few pitchers that have consistently slower fastballs. The pitch speed is the speed at the release point: the ball will be going 10% slower when it crosses the

TABLE 16.2

Typical Values for Major League Pitches

Type of Pitch	Initial Speed (mph)	Initial Speed (m/s)	Spin Rate (rpm)	Spin Rate (rps)	Rotations between Pitcher's Release and the Point of Bat–Ball Contact
Fastball	85–95	38–42	1200	20	8
Slider	80–85	36–38	1400	23	10
Curveball	70–80	31–36	2000	33	17
Changeup	60–70	27–31	400	7	4
Knuckleball	60–70	27–31	30	½	¼

plate. In this chapter, the equations are general and should apply to many types of spinning balls. However, whenever we give specific numerical values they are (unless otherwise stated) for major league baseball.

The earliest empirical equation for the transverse force on a spinning object moving in a fluid is the Kutta–Joukowski lift theorem

$$\mathbf{L} = \rho \mathbf{U} \times \Gamma \tag{16.2}$$

where

L is the lift force per unit length of cylinder
ρ is the fluid density
U is the fluid velocity
Γ is the circulation around the cylinder
L, **U**, and **Γ** are vectors

When this equation is tailored for a baseball [10, pp. 77–81], we get the magnitude of the spin-induced force acting perpendicular to the direction of flight

$$F_{\text{perpendicular}} = F_{\text{Magnus}} = 0.5\ \rho \pi r_{\text{ball}}^3 \omega\, v_{\text{ball}} \tag{16.3}$$

where ω is the spin rate. This is usually called the Magnus force. This force can be decomposed into a force lifting the ball up and a lateral force pushing it sideways.

$$F_{\text{upward}} = 0.5\, \rho \pi r_{\text{ball}}^3 \omega\, v_{\text{ball}} \sin \text{VaSa} \tag{16.4}$$

where VaSa is the angle between the vertical axis and the spin axis (Figures 16.4 and 16.8). The magnitude of the lateral force is

$$F_{\text{sideways}} = 0.5\, \rho \pi r_{\text{ball}}^3 \omega\, v_{\text{ball}} \cos \text{VaSa} \tag{16.5}$$

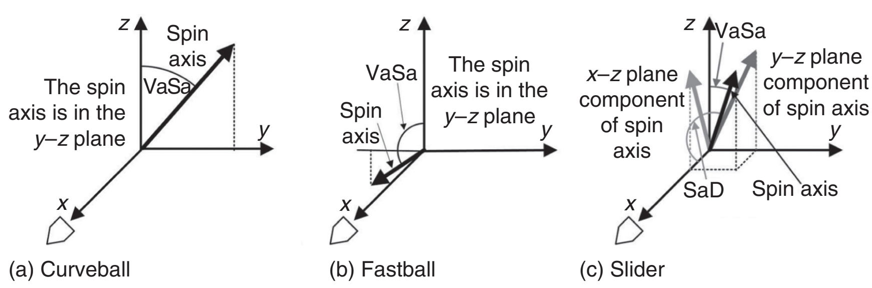

FIGURE 16.8 (See color insert following page xxx.)
Rectangular coordinate system and illustration of the angles VaSa and SaD for (a) curveball, (b) three-quarter arm fastball, and (c) slider all thrown by a right-handed pitcher. The origin is the pitcher's release point. For the curveball, the spin axis is in the y–z plane. For the fastball, the spin axis is also in the y–z plane, but it is below the y-axis. For the slider, the spin axis has components in both the y–z and x–z planes. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2006.)

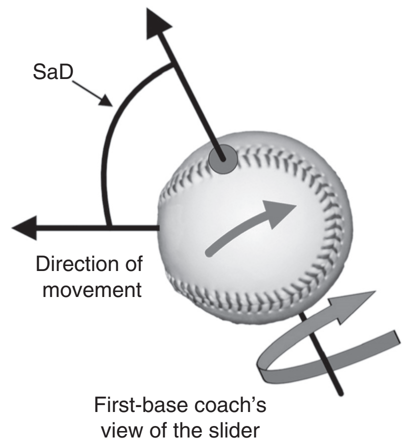

FIGURE 16.9 (See color insert following page xxx.) The first-base coach's view of a slider thrown by a right-handed pitcher. This illustrates the definition of the angle SaD. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2007.)

Finally, if the spin axis is not perpendicular to the direction of motion (as in the case of the slider), the magnitude of the cross product of these two vectors will depend on the angle between the spin axis and direction of motion; this angle is called SaD (Figures 16.8 and 16.9). In aeronautics, it is called the angle of attack.

$$F_{\text{lift}} = 0.5\,\rho\pi r_{\text{ball}}^3\omega\, v_{\text{ball}} \sin \text{VaSa} \sin \text{SaD} \tag{16.6}$$

$$F_{\text{lateral}} = 0.5\,\rho\pi r_{\text{ball}}^3\omega\, v_{\text{ball}} \cos \text{VaSa} \sin \text{SaD} \tag{16.7}$$

During the pitch, gravity is continuously pulling the ball downward, which changes the direction of motion of the ball by 5° to 10° during its flight. However, the ball acts like a gyroscope, so the spin axis does not change. This means that, for a slider, the angle SaD increases and partially compensates for the drop in ball speed in Equations 16.6 and 16.7.

16.1.5 Comparison of the Slider and Curveball

Let us now compare the magnitude of this lateral spin-induced deflection force (Equation 16.7) for two specific pitches, namely the slider and the curveball. The magnitude of the lateral spin-induced deflection of the slider is less than that of a curveball for the following four reasons:

1. For the curveball, the angle between the spin axis and the direction of motion (SaD) is around 85°. For the slider, it is around 60°. The magnitude of the cross product is proportional to the sine of this angle. Therefore, the slider's deflection force is less than the curveball's by the ratio $\frac{\sin 60}{\sin 85}$: the slider force equals 0.87 times the curveball force. The angle between the vertical axis and the spin axis (VaSa) has no effect because it is about the same for the slider and the curveball.

2. Curveball spins at up to 33 revolutions per second (rps) and the slider probably spins around 23 rps [16], and hence the slider's deflection force is smaller because of its slower rotation. Thus, the slider force equals 0.7 times the curveball force.
3. Deflection force also depends on the speed of the pitch. Assume a 75 mph (34 m/s) curveball and an 85 mph (38 m/s) slider: the slider force equals 1.13 times the curveball force.

   Therefore, for the three effects of this example, the total slider force equals 0.69 times the curveball force.
4. Furthermore, the curveball is slower, so it is in the air longer. Therefore, the deflection force has longer to operate and the total deflection due to this effect is greater. An 85 mph (38 m/s) slider travels from the pitcher's release point, 5 ft (1.5 m) in front of the rubber, to the point of bat–ball collision, 1.5 ft (0.5 m) in front of the plate, in 453 ms, whereas a 75 mph (34 m/s) curveball is in the air for 513 ms: squaring these durations gives a ratio of 0.78. The total deflection is proportional to total force times duration squared: therefore, the ratio deflection of the slider with respect to the curveball is

$$\begin{aligned}&(\text{ratio-force}_{\text{spin axis}})(\text{ratio-force}_{\text{spin rate}})(\text{ratio-force}_{\text{speed}})\\&\quad(\text{ratio-durations squared}) = (0.87)(0.7)(1.13)(0.78) = 0.54\end{aligned}$$

In summary, the magnitude of the lateral spin-induced deflection of the slider is about half that of the curveball.

The screwball (sometimes called a ''fadeaway'' or ''in-shoot'') was made popular in the early 1900s by Christy Mathewson and Mordecai ''Three Fingered'' Brown and was repopularized by the left-hander Carl Hubbell in the 1930s. Therefore, we show it from the left-hander's perspective in Figure 16.5. Of the pitches shown in Figures 16.4 and 16.5, it is the least used, in part, because the required extended pronation of the hand strains the forearm and elbow. At release, the fingers are on the inside and top of the ball. The deflection of the left-hander's screwball is the same as the deflection of a right-hander's slider. The spin of the screwball is basically like that of a slider, so its deflection will be less than that of a curveball for the reasons given above.

The direction of deflection of these pitches is variable depending on the direction of the spin axis. The direction of this axis varies with the angle of the arm during delivery and the position of the fingers on the ball at the time of release. By controlling his or her arm angle and finger positions, the pitcher controls the direction of deflection.

16.1.6 Vertical Deflection

Tables 16.3 and 16.4 show the magnitude of the spin-induced drop for three kinds of pitches at various speeds, as determined by our simulations. Our

TABLE 16.3

Gravity- and Spin-Induced Drop (with English Units)

Pitch Speed and Type	Spin Rate (rpm)	Duration of Flight (ms)	Drop due to Gravity (ft)	Spin-Induced Vertical Drop (ft)	Total Drop (ft)
95 mph fastball	−1200	404	2.63	−0.91	1.72
90 mph fastball	−1200	426	2.92	−0.98	1.94
85 mph slider	+1400	452	3.29	+0.74	4.03
80 mph curveball	+2000	480	3.71	+1.40	5.11
75 mph curveball	+2000	513	4.24	+1.46	5.70

baseball trajectory simulator includes the effects of lift and drag due to spin on the ball [10,11,17,18]. Looking at one particular row, a 90 mph (40.2 m/s) fastball is in the air for 426 ms, so it drops 2.92 ft (0.89 m) due to gravity ($\frac{1}{2}\,gt^2$, where the gravitational constant g is 32.2 ft/s^2 (9.8 m/s^2) and t is the time from release until the point of bat–ball collision). But the backspin lifts this pitch 0.98 ft (0.3 m), producing a total drop of 1.94 ft (0.59 m) as shown in Tables 16.3 and 16.4. In the spin rate column, negative numbers are backspin and positive numbers are topspin. In the spin-induced vertical drop column, negative numbers mean the ball is being lifted up by the Magnus force. All of the pitches in Tables 16.3 and 16.4 were launched horizontally—that is, with a launch angle of zero. The angle VaSa was also set to zero (simulating an overhand delivery): therefore, pitches thrown with a three-quarter arm delivery would have smaller spin-induced deflections than given in Tables 16.3 and 16.4.

Vertical misjudgment of the potential bat–ball impact point is a common cause of batters' failure to hit safely [3,13]. The vertical differences between the curveballs and fastballs in Tables 16.3 and 16.4 are greater than 3 ft (1 m), whereas the difference between the two speeds of fastballs is around 3 in. (7 cm) and the difference between the two speeds of curveballs is around 7 in. (18 cm). However, the batter is more likely to make a vertical error because speed has been misjudged than because the kind of pitch has been misjudged [3,13]. A vertical error of as little as one-third of an inch (8 mm) in the batter's swing will generally result in a failure to hit safely [3,13], as is shown in Section 16.3.

TABLE 16.4

Gravity- and Spin-Induced Drop (with SI Units)

Pitch Speed and Type	Spin Rate (rad/s)	Duration of Flight (ms)	Drop due to Gravity (m)	Spin-Induced Vertical Drop (m)	Total Drop (m)
42.5 m/s fastball	−126	404	0.80	−0.28	0.52
40.2 m/s fastball	−126	426	0.89	−0.30	0.59
38.0 m/s slider	+147	452	0.95	+0.23	1.23
35.8 m/s curveball	+209	480	1.13	+0.43	1.56
33.5 m/s curveball	+209	513	1.29	+0.45	1.74

The spin on the pitch also causes a horizontal deflection of the ball. In ''deciding'' whether to swing, the horizontal deflection is more important than the vertical, because the umpire's judgment with respect to the corners of the plate has more precision than his or her judgment regarding the top and bottom of the strike zone. However, after the batter has decided to swing and is trying to ''track and hit'' the ball, the vertical deflection becomes more important.

The right-hand rules for the lateral deflection of a spinning ball also apply to the batted ball, except it is harder to make predictions about the magnitude of deflection because we have no data about the spin on a batted ball. The right-hand rules can be applied to tennis, where deflections are similar to baseball, but not to American football, because spin-induced deflections of a football are small [19]. A professional quarterback throws a pass at around 80 mph with 12 rps spin.

16.1.7 Modeling Philosophy

Although our equations and discussion might imply great confidence and precision in our numbers, it is important to note that our equations are only models. The Kutta–Joukowski equation and subsequent derivations are not theoretical equations, they are only approximations fit to experimental data. There are more complicated equations for the forces on a baseball (e.g., see [20–25]). Furthermore, there is much that we did not include in our model. We ignored the possibility that air flowing around certain areas of the ball might change from turbulent to laminar flow en route to the plate. Our equations did not include effects of shifting the wake of turbulent air behind the ball. En route to the plate, the ball loses 10% of its linear velocity and 2% of its angular velocity: we did not include this reduction in angular velocity. We ignored the difference between the center of gravity and the geometrical center of the baseball [9]. We ignored possible differences in the moments of inertia of different balls. We ignored the precession of the spin axis. In computing velocities due to bat–ball collisions, we ignored deformation of the ball and energy dissipated when the ball slips across the bat surface. Finally, as we have already stated, we treated the drag coefficient as a constant.

We used a value of 0.5 for the drag coefficient, C_d. However, for speeds over 80 mph this drag coefficient may be smaller [10, p. 157; 20,23,24]. There are no wind-tunnel data showing the drag coefficient of a spinning baseball over the range of velocities and spin rates that characterize a major league pitch. Sawicki et al. [22] summarize data from a half-dozen studies of spinning baseballs, nonspinning baseballs, and other balls and showed C_d between 0.15 and 0.5. In most of these studies, the value of C_d depended on the speed of the airflow. In the data of Ref. [25], the drag coefficient can be fit with a straight line of $C_d = 0.45$, although there is considerable scatter in these data. The drag force causes the ball to lose about 10% of its speed en route to the plate. The simulations of Ref. [26] also studied this loss in speed.

Data shown in Figure 9 of Ref. [26] for the speed lost en route to the plate can be nicely fitted with $PercentSpeedLost = 20\ C_d$, which implies $C_d = 0.5$.

It is somewhat surprising that given the multitude of modern computer camera pitch-tracking devices such as the QuesTec system, the best-published experimental data for the spin rate of different pitched baseballs come from Selin's cinematic measurements of baseball pitches [16]. And we have no experimental data for the spin on the batted ball. Table 16.2 summarizes our best estimates of speed and spin rates for most popular major league pitches.

There is uncertainty in the numerical values used for the parameters in our equations. However, the predictions of the equations match baseball trajectories quite well. When better experimental data become available for parameters such as C_d and spin rate, then values of other parameters will have to be adjusted to maintain the match between the equations and actual baseball trajectories.

The value of this present study lies in comparisons rather than absolute numbers. Our model emphasizes that the right-hand rules show the direction of the spin-induced deflections of a pitch. The model provides predictive power and comparative evaluations relative to the behavior of all kinds of pitches.

Stark [27] explained that models are ephemeral: they are created, they explain a phenomenon, they stimulate discussion, they foment alternatives, and then they are replaced by new models. When there are better wind-tunnel data for the forces on a spinning baseball, then our equations for the lift and drag forces on a baseball will be supplanted by newer parameters and equations. But we think our models, based on the right-hand rules showing the direction of the spin-induced deflections, will have permanence: they are not likely to be superseded.

16.1.8 Somatic Metaphors of Pitchers

A pitcher uses his or her hand as a metaphor for the ball when asked to demonstrate the trajectory of a particular kind of pitch (such as a screwball). But he or she derives a mental model of a specific pitch from the feelings of arm angle and his or her fingers on the ball as the pitch is being released. By imagining slight shifts in these sensations, the pitcher can create subtly differing models that can provide pitch variability to his or her repertoire. For example, he or she might model the screwball with fingers on top of the ball when it is released (resulting in a downward deflection) or with fingers on the side of the ball (resulting in a flatter deflection).

The batter finds it hard to distinguish subtle differences in the spin direction of a specific kind of pitch. For example, a 95 mph (42.5 m/s) fastball thrown directly overhand looks much like a 95 mph fastball thrown with the arm angle lowered by 20°. The vertical difference in the potential bat–ball contact point, however, is significant. For the 95 mph fastball with a 1200 rpm backspin shown in Tables 16.3 and 16.4, the pitch thrown with the lower arm angle would drop about three-quarters of an inch (2 cm) farther than the overhand pitch. Three-quarters of an inch is bigger than the vertical sweet spot. Mental models of pitch differences allow the pitcher to take advantage of the batter's difficulty in recognizing a wide variety of spin directions and detecting small shifts in arm angle.

16.1.9 Summary

Somatic metaphors are pervasive in everyday life, so it is not surprising to find that baseball pitchers make use of these modeling devices in their work. We have shown how a pair of widely used engineering metaphors, the right-hand rules, provides a formalized approach to describing the pitchers' mental models, allowing prediction of the deflection direction of each pitch. Besides describing the behavior of the pitched ball, these rules can also be used in describing the deflection direction of the batted ball. To determine the direction of deflection of the pitched or the batted ball, point the thumb of your right hand in the direction of the Spin axis and your index finger in the Direction of motion of the ball; your middle finger will indicate the direction of the Spin-induced deflection (SaD Sid).

16.2 Bat–Ball Collisions

16.2.1 Sweet Spot of the Bat

For skilled batters, we assume that most bat–ball collisions occur near the sweet spot of the bat, which is, however, difficult to define precisely. The horizontal sweet spot has been defined as the center of percussion (CoP), the node of the fundamental bending vibrational mode, the antinode of the hoop mode, the maximum energy transfer area, the maximum batted-ball speed area, the maximum CoR area, the minimum energy loss area, the minimum sensation area, and the joy spot [2,28]. Let us now examine each of these definitions.

1. **Center of percussion**. For most collision points, when the ball hits the bat it produces a translation of the bat and a rotation of the bat. However, if the ball hits the bat at the center of mass there will be a translation but no rotation. Whereas, if the bat is fixed at a pivot point and the ball hits the bat at the CoP for that pivot point, then there will be a rotation about that pivot point but no translation (and therefore no sting on the hands). The pivot point and the CoP for that pivot point are conjugate points, because if instead the bat is fixed at the CoP and the ball hits the pivot point then there will be a pure rotation about the CoP. The CoP and its pivot point are related by the following equation derived by Sears et al. [29], where the variables are defined in Figure 16.10:

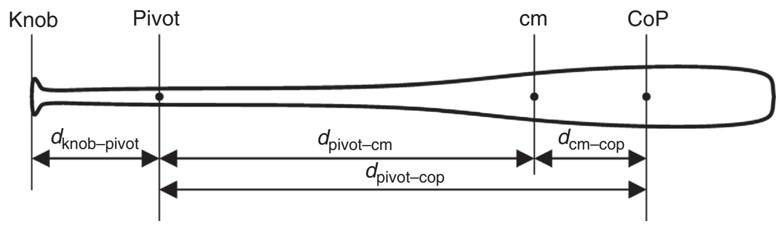

FIGURE 16.10
Definition of distances on a bat. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2001.)

$$d_{\text{pivot-cop}} = \frac{I_{\text{pivot}}}{m_{\text{bat}} d_{\text{pivot-cm}}} \tag{16.8}$$

The CoP is not one fixed point on the bat. There is a different CoP for every pivot point. If the batter chokes up on the bat, the pivot point (and consequently the CoP) will change. In fact, the pivot point might even change during an individual swing. In this chapter, we assume that the pivot point is 6 in. (15 cm) from the knob.

There are three common experimental methods for determining the CoP.

Method 1: Pendular motion: Hang a bat at a point 6 in. (15 cm) from the knob with 2 or 3 ft (1 m) of string. Hit the bat with an impact hammer. Hitting it off the CoP will make it flop like a fish out of water, because there is a translational force and a rotational force at the pivot point. Hitting it near the CoP will make it swing like a pendulum (as shown in Figures 12 and 13 of Ref. [29]).

Method 2: Toothpick pivot: Alternatively, you can pivot the bat on a toothpick through a hole at the pivot point 6 in. from the knob and strike the bat at various places. When struck near the CoP for that pivot point the toothpick will not break. At other places, the translational forces will break the toothpick.

Method 3: Equivalent pendulum: A third method for measuring the distance between the pivot point and the CoP is to make a pendulum by putting a mass equal to the bat's mass on a string and adjusting its length until the pendulum's period and the bat's period are the same. This method has the smallest variability.

2. **Node of the fundamental mode**. The node of the fundamental bending vibrational mode is the area where this vibrational mode (roughly between 150 and 200 Hz for a wooden bat) of the bat has a null point [20,30–33]. To find this node, grip a bat about 6 in. from the knob with your fingers and thumb. Lightly tap the barrel at various points with an impact hammer. The area where you feel no vibration and hear almost nothing (except the secondary

vibration crack or ping at 500 to 800 Hz) is the node. A rubber mallet could be used in place of an impact hammer: the point is, the hammer itself should not produce any noise. The antinode of the third bending vibrational mode may also be important [34].

3. **Antinode of the hoop mode**. For hollow metal and composite baseball and softball bats, there is another type of vibration, called a hoop vibration. The walls of a hollow bat deform during a bat–ball collision. The walls are crushed in and then bounce back out. This vibration can be modeled as a hoop or a ring around the bat; this ring deforms like the vertical cross-sectional area of a water drop falling from a faucet; first the water drop is tall and skinny, in free fall it is round, and when it hits the ground it becomes short and fat. The location of the antinode of the first hoop mode is another definition of the sweet spot [34,35].
4. **Maximum energy transfer area**. A collision at the maximum energy transfer area transfers the most energy to the ball [36]. This derivation is reproduced in Ref. [10]. This definition says that the best contact area on the bat is that which loses the least amount of energy to bat translation, rotation, vibration, etc. This would be a more useful definition if it specified maximum ''useful'' energy transfer—the useful energy is that which moves the ball in the same direction as the trajectory of the bat. In this definition, energy stored in the spin of the ball is not useful.
5. **Maximum batted-ball speed area**. There is an area of the bat that produces the maximum batted-ball speed [32,33,37,38]. This area is about 5 or 6 in. from the end of the barrel for wooden bats and about 7 in. from the end of the barrel for aluminum bats [32,33]. This would be a more useful definition if it specified ball velocity rather than ball speed (since the bat is a three-dimensional object).
6. **Maximum coefficient of restitution area**. The CoR is commonly defined as the ratio of the relative speed after a collision to the relative speed before the collision. In our studies, the CoR is used to model the energy transferred to the ball in a collision with a bat. If the CoR were 1, then all the original energy would be recovered in the motion of the system after impact. But if there were losses due to energy dissipation or energy storage, then the CoR would be less than 1. For example, in a bat–ball collision there is energy dissipation: both the bat and the ball increase slightly in temperature. Duris and Smith [46] said in their presentation that 100 bat–ball collisions in rapid succession raised the temperature of a softball by 10°F. Also both the bat and the ball store energy in vibrations. Not all of this energy will be transferred to the ball. (For now, we ignore the kinetic energy stored in the ball's spin.) The maximum CoR area is the area that produces the maximum CoR for a bat–ball collision [32,36].

7. **Minimum energy loss area**. There is an area that minimizes the total (translation plus rotation plus vibration) energy lost in the handle. This area depends on the fundamental bending mode, the second mode, and the CoP [39].
8. **Minimum sensation area**. For most humans, the sense of touch is most sensitive to vibrations between 200 and 400 Hz. For each person there is a collision area on the bat that would minimize these sensations in the hands [40].
9. **Joy spot**. Finally, Williams and Underwood [41] stated that hitting the ball at the joy spot makes you the happiest. The joy spot was centered 5 in. (13 cm) from the end of the barrel.

These nine areas are different, but they are close together. We group them together and refer to this region as the sweet spot. We measured a large number of bats (youth, adult, wood, aluminum, ceramic, titanium, etc.) and found that the sweet spot was 15%–20% of the bat length from the barrel end of the bat. This finding is in accord with Refs. [20,30–32,39–42] as well as Worth Sports Co. (personal communication) and Easton Aluminum Inc. (personal communication). In our ideal bat weight experiments [4,43] and our variable moment of inertia experiments [2] for adult bats, the center of the sweet spot was defined to be 5 in. (13 cm) from the barrel end of the bat.

It does not make sense to try getting greater precision in the definition of the sweet spot, because the concept of a sweet spot is a human concept, and it probably changes from human to human. For one example, in calculating the CoP, the pivot point of the bat must be known and this changes from batter to batter, and it may even change during the swing of an individual batter.

Table 16.5 shows general properties for a standard Hillerich and Bradsbury Louisville Slugger wooden C243 pro stock 34 in. (86 cm) bat

TABLE 16.5

Parameters for a C243 Wooden Bat

Stated Length (in.)	**34**
Period (s)	1.634
Mass (kg)	0.905
I_{knob} (kg m^2)	0.342
I_{pivot} (kg m^2)	0.208
I_{cm} (kg m^2)	0.048
Measured $d_{\text{knob–cm}}$ (cm)	57
Measured $d_{\text{knob–cop}}$ (cm)	69
Calculated $d_{\text{knob–cop}}$ (cm)	69
Measured $d_{\text{pivot–cop}}$ (cm)	55
Calculated $d_{\text{pivot–cop}}$ (cm)	54
Measured $d_{\text{knob–firstNode}}$ (cm)	67

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TABLE 16.6

Distance in Centimeters from the Barrel End to the Center of the Sweet Spot for a 34 in. Wooden Bat

Definition of Sweet Spot	This Study of a C243 Wooden Bat	References
Center of percussion for a 15 cm pivot point[a]	16 calculated	16.5 [38][b]
	18 experimental method 1:	More than 15 [47]
	15 experimental method 2:	17 [36]
	14 experimental method 3:	
Maximum energy transfer area		20 [36]
Maximum batted-ball speed area		14 [32,33]
		17 [38]
Maximum coefficient of restitution area		15 [32]
Node of fundamental vibration mode[c]	18 measured	17 [33]
		17 [38]
		17 [39]
Minimum sensation area		17 [40]
Minimum energy area		15 [39]
		15–18 [31]
Joy spot		13 [41]

[a] The center of percussion for a uniform rod would be 15 cm from the end [48]. This is a lower limit for a bat.

[b] Ref. [38] used a 33 in. bat and their CoP was 16/84 = 19% from the barrel end: scaling for a 34 in. bat yields 16.5 cm.

[c] The node of the fundamental vibration mode of an open-ended pipe is 0.224 times the length. For a 34 in. pipe, it would be 19 cm from the end. This is an upper limit for a bat.

with the barrel end cupped out to reduce weight. Table 16.6 shows sweet spot parameters for this and similar 34 in. wooden bats. These modern scientific methods of calculating the center of the sweet spot of the bat are all only a few centimeters above the true value given by Williams a quarter century ago. Table 16.7 shows several other parameters for a variety of commercially available bats.

There is no sweet spot of the bat: however, there is a sweet area and for a 34 in. wooden bat, it is 5 to 7 in. (13 to 18 cm) from the barrel end of the bat. We presented nine definitions for the sweet spot of the bat. Some of these definitions had a small range of experimentally measured values (e.g., 1 cm for the node of the fundamental vibration mode), whereas others had a large range of experimentally measured values (e.g., 10 cm for the maximum batted-ball speed area). But of course, none of these definitions have square sides. They are all bowl shaped. So the width depends on how far you allow the parameter to decline before you say that you are out of the sweet area. In general, the sweet area is about 2 in. wide. Our survey of retired major league batters confirmed that the sweet spot of the bat is about 2 in. (5 cm) wide. Therefore, most of the sweet spot definitions of this chapter fall within this region. In summary, recent scientific analyses have validated William's statement that the sweet spot of the bat is an area 5 to 7 in. from the end of the barrel.

TABLE 16.7

Properties of Typical Commercially Available Bats

League	Stated Weight (oz)	Length (in.)	Period (s)	Mass (kg)	Distance from the Knob to Center of Mass, $d_{\mathrm{knob-cm}}$ (m)	Moment of Inertia with Respect to the Knob, I_{knob} (kg m^2)	Moment of Inertia with Respect to the Center of Mass, I_{cm} (kg m^2)
Tee ball	17	25	1.420	0.478	0.346	0.083	0.026
Little League	22	31	1.570	0.634	0.448	0.174	0.047
High school	26	32	1.669	0.764	0.510	0.269	0.070
Softball	23	33	1.584	0.651	0.477	0.193	0.045
Softball, end loaded	26	34	1.667	0.731	0.505	0.255	0.069
Softball, end loaded	29	34	1.674	0.810	0.506	0.285	0.078
Major league, R161 (wood)	32	34	1.654	0.920	0.571	0.356	0.056
Major league, C243 (wood)	32	34	1.634	0.905	0.570	0.342	0.048

16.2.2 Coefficient of Restitution

The CoR is commonly defined as the ratio of the relative speed after a collision to the relative speed before the collision [10,29,32]. In our studies, the CoR is used to model the energy transferred to the ball in a collision with a bat. If the CoR were 1.0, then all the original energy would be recovered in the motion of the system after impact. But if there were losses due to energy dissipation or energy storage, then the CoR would be less than 1.0. For example, in a bat–ball collision there is energy dissipation: both the bat and the ball increase slightly in temperature. Also both the bat and the ball store energy in vibrations. This energy is not available to be transferred to the ball and therefore the ball velocity is smaller. (We ignore the kinetic energy stored in the ball's spin.)

The CoR depends on many things including the shape of the object that is colliding with the ball. When a baseball is shot out of an air cannon onto a flat wooden wall, most of the ball's deformation is restricted to the outer layers: the cowhide cover and the four yarn shells. However, in a high-speed collision between a baseball and a cylindrical bat, the deformation penetrates into the cushioned cork center. This allows more energy to be stored and released in the ball and the CoR is higher. In our model, the CoR for a baseball–bat collision is 1.17 times the CoR of a baseball–wall collision. The CoR also depends on the speed of the collision. Our computer programs use the following equations for the CoR: for an aluminum bat and a softball:

$$CoR = 1.17\,(0.56 - 0.001\ CollisionSpeed) \tag{16.9}$$

for a wooden bat and a baseball

$$CoR = 1.17\,(0.61 - 0.001\ CollisionSpeed) \tag{16.10}$$

where *CollisionSpeed* (the sum of the magnitudes of the pitch speed and the bat speed) is in miles per hour. These equations come from unpublished data provided by Jess Heald of Worth Sports Co. and they assume a collision at the sweet spot. Our baseball CoR equation is in concordance with data from six studies summarized in a report to the NCAA [44]: $CoR = 1.17\ (0.57 - 0.0013\ CollisionSpeed)$.

The CoR also depends on where the ball hits the bat, because different locations produce different vibrations in the bat [20,30,32,33]. Increasing the humidity of the ball from 10% to 90% decreases the CoR by roughly 15%. Ball temperature affects the CoR [20,45]. Bat temperature also affects the CoR: so bat warmers in the dugout would increase the CoR. But we will not consider these complexities in this chapter.

In the past, the CoR of a baseball–bat collision was mostly a property of the ball, because a wooden bat does not deform during a bat–ball collision. But hollow metal and composite baseball and softball bats do deform during the collision; thus, they play an important part in determining the CoR. During a collision, energy is stored in the ball and in the bat. Most of the energy stored in the ball is lost. This energy loss is modeled with the CoR. If the CoR is half, then three-fourth of the energy is lost (because kinetic energy is proportional to velocity squared). Most of the energy stored in the bat is not lost, but is transferred to the ball. This increases the batted-ball speed. This matching of the bat to the ball to increase batted-ball speed is called the trampoline effect [34]. Because most of the energy stored in the ball is lost and most of the energy stored in the bat is returned, the batter would prefer to have energy stored in the bat rather than in the ball. A hard (or stiff) ball will deform the bat more and therefore store more energy in the bat, which, by the above argument, will increase batted-ball speed. Therefore, the hardness (or stiffness) of the ball becomes another regulated parameter. Today, softballs are typically marked with a CoR number and a stiffness number. The stiffness is the amount of slowly applied force that is required to deform a softball by ¼ inch (0.64 cm) [46].

16.2.3 Performance Criterion

In most engineering studies, the most important decision is choosing the performance criterion. For a batter hitting a ball, what is the most important performance criterion? Kinetic energy imparted to the ball? Momentum imparted to the ball? Batted-ball speed? Accuracy? Launch angle? Batted-ball spin rate? Batted-ball spin axis? Efficiency of energy transfer? or Distance from the plate where the ball hits the ground? For most studies in the baseball literature, the performance criterion was maximizing batted-ball speed. Where in baseball would other performance criteria be more appropriate?

In calculating knockdown power, kinetic energy would be appropriate. The Colt 0.45 automatic pistol was designed for battles in the Philippines in the early years of the twentieth century, with the performance criterion of ''Knock down the charging warrior before he or she can chop off your head with a machete.'' The existing 0.38 would kill him, but he or she would chop off your head before he or she would die. A solution for this problem was the 0.45 caliber munition with a muzzle kinetic energy of 370 ft-lb_m (502 J). (The kinetic energy of bullets is given in units of foot-pounds, but the pounds are not pounds-force, rather they are pounds-mass.) So 1 ft-$lb_m = 1.36$ J. In contrast, a baseball traveling at 97 mph (43 m/s) has 100 ft-lb_m (136 J) of kinetic energy. This explains why a hit-batter can be hurt, but not knocked down by a pitch.

As an aside, the energy stored in the spin of a baseball is $KE_{spin} = \frac{I\omega^2}{2} = \frac{mr_{ball}^2\omega^2}{5}$. Substituting in nominal values for a baseball spinning at 1200 rpm yields $KE_{spin} = \frac{0.145 \times 0.0014 \times 15.791}{5} = 0.6$ J (0.5 ft-lb_m), which is much lesser than the translational energy.

Here are some potential performance criteria for a pitcher: (1) minimize the number of pitches per inning, by getting the hitter to hit an early pitch for a grounder (this would reduce the batter's opportunities to learn the pitches and lessen pitcher fatigue), (2) minimize the number of runs, (3) maximize batter intimidation, and (4) generate impressive statistics (e.g., strikeouts, wins, ERA, saves) that would generate high salaries.

16.2.4 Vertical Size of the Sweet Spot

We need a model for batting success that shows the relative importance of bat weight, bat speed, launch angle, bat shape, and coefficient of friction. These are all under the batter's control. We [3,13] developed a new performance criterion: the probability of getting a hit. The old performance criterion of maximizing batted-ball speed works well for home runs, but only 4% of batted balls in play are home runs.

We now introduce a new criterion for the batted ball, the distance from the plate where the ball first hits the ground. Assume that the batter wants to hit a line drive. He or she wants the ball to clear the infielders without bouncing, and to hit the grass in front of the outfielders. Thus, a major league baseball player wants the ball to hit the ground between 120 and 240 ft (37 to 73 m) from the plate. These numbers were given in Table 16.1.

We now make the following assumptions. The batter is using a Louisville Slugger C243 wooden bat and is hitting a regulation baseball. The pitch speed is 85 mph (38 m/s). The speed of the sweet spot of the bat is 60 mph (27 m/s): this is the average value for the San Francisco Giants measured by Bahill and Karnavas [43]. These speeds would produce a CoR of 0.54. The bat weighs 32 oz (0.91 kg) and the ball weighs 5.125 oz

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(0.145 kg). We can put these data into the following equation from Ref. [2] to get a batted-ball speed of 106 mph (47 m/s), which is a reasonable value:

$$v_{\text{ball-after}} = v_{\text{ball-before}} + \frac{(1+\text{CoR})(v_{\text{bat-before}} - v_{\text{ball-before}})}{1 + \dfrac{m_{\text{ball}}}{m_{\text{bat}}} + \dfrac{m_{\text{ball}} d^2_{\text{cm-ss}}}{I_{\text{cm}}}} \tag{16.11}$$

This performance criterion is used in the next section to define the vertical sweet spot of the bat.

16.3 Model for Bat–Ball Collisions

Baseball and softball batters swing a narrow cylinder with the axis more or less parallel with the ground. Thus, the transverse curvature of the bat's face (hitting surface) is a vertical curvature. In combination with the vertical offset of the bat and ball trajectories, this vertical curvature strongly influences the ball's vertical launch velocity, angle, and spin rate. These launch characteristics can be included in a vector describing a specific point on the bat's face; a vector field can specify the launch characteristics of all the points on the face. Each vector determines the batted ball's behavior—the distance it travels in the air until it first strikes the ground (range), how long it stays in the air (hang time), and, for ground balls, the time taken for the ball to reach the positional arc of infielders (ground time).

The set of success probabilities associated with a specific vertical arc on the bat's face is called the vertical sweetness gradient of that arc. The face's vector field represents sweetness gradients in both the longitudinal (horizontal) and transverse (vertical) dimensions of the bat. However, we restrict our current discussion to vertical collision considerations and the radial placement of the ball in play in fair territory.

We integrated many models as shown in Figure 16.11. One of the input parameters in the overall model is the offset between the bat and the ball. This offset is defined in Figure 16.12. The basic principle of this model is that we break up the bat and ball velocities into normal and tangential components. We apply conservation of energy. And then we apply conservation of linear and angular momentum. This technique is suggested in Figure 16.13.

Finally, Figure 16.14 shows the full model. It illustrates the initial vertical configuration of the bat and ball at the instant of collision. The initial parameters of the collision are:

1. Initial velocity vector of bat's contact point ($v_{\text{bat,0}}$)
2. Initial normal component of the bat's velocity vector ($v_{\text{bat,0,n}}$)
3. Initial tangential component of the bat's velocity vector ($v_{\text{bat,0,t}}$)
4. Initial velocity vector of ball ($v_{\text{ball,0}}$)

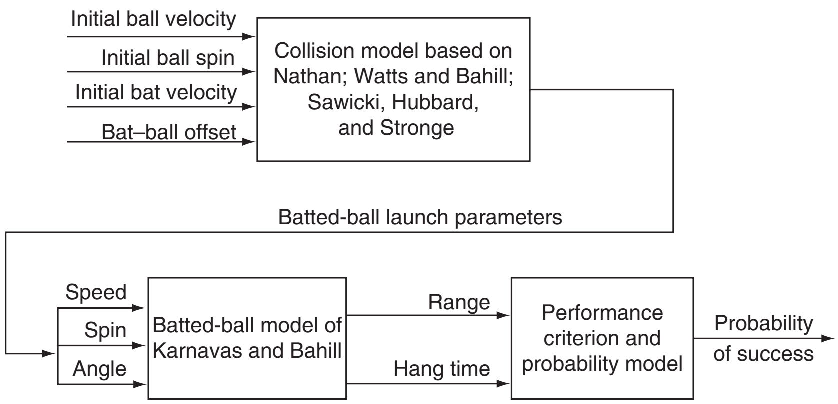

FIGURE 16.11
Our model used components from several other models. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

5. Initial normal component of the ball's velocity vector ($v_{\text{ball,0,n}}$)
6. Initial tangential component of the ball's velocity vector ($v_{\text{ball,0,t}}$)
7. Bat–ball offset distance (D) from the ball's center perpendicular to the trajectory plane of the bat's transverse center
8. Vertical angle (θ) between the line connecting ball and bat centers (line of centers) and the horizontal plane ($z = 0$)
9. Vertical angle (γ) between the horizontal plane and the ball's trajectory plane

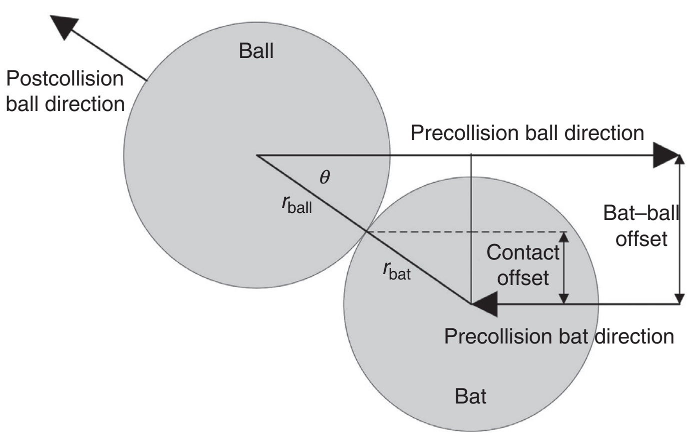

FIGURE 16.12
Definition of the bat–ball offset. (This figure does not show the effect that pitch spin has on the postcollision ball direction. For most collisions, the ball is going down at a 10° angle and the bat is going up at a 10° angle. These angles are not shown in this figure.) (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

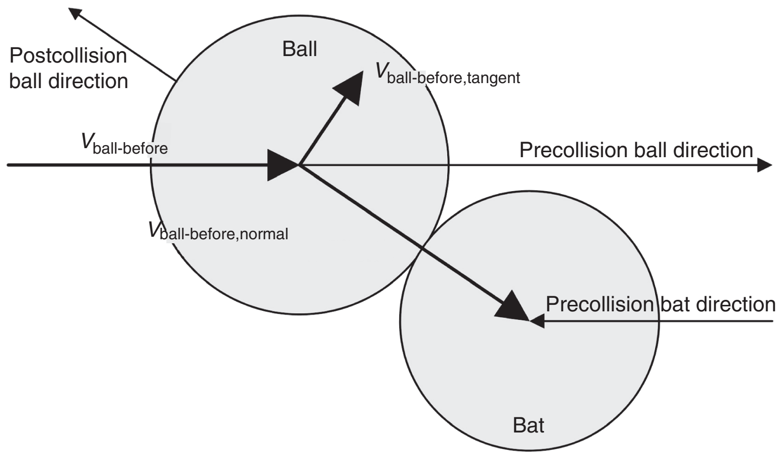

FIGURE 16.13
The bat and ball velocities are decomposed into normal and tangential components. (This figure ignores the spin of the ball.) (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

10. Vertical angle (ψ) between the horizontal plane and the bat's trajectory plane
11. Mass of bat (m_{bat})
12. Mass of ball (m_{ball})
13. Radius of bat at contact point (r_{bat})
14. Radius of ball (r_{ball})

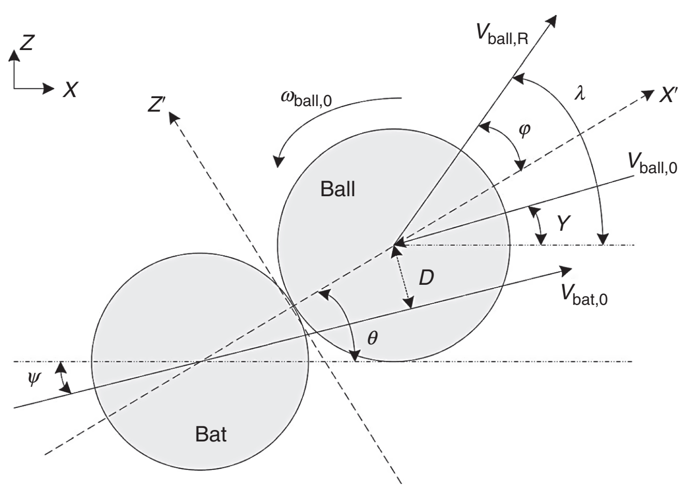

FIGURE 16.14
Initial vertical configuration of the bat–ball collision. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

15. Coefficient of restitution (CoR) of the bat–ball impact
16. Coefficient of friction (μ) during bat–ball impact
17. Angular velocity ($\omega_{\text{ball,0}}$) of the pitch

The characteristics m_{ball}, r_{ball}, CoR, and μ will be considered constants. The values of CoR and μ must be derived empirically.

The resultant vectors and angles used to calculate launch velocity and angle are:

1. Resultant velocity vector of bat's contact point ($v_{\text{bat,R}}$)
2. Resultant normal component of the bat's velocity vector ($v_{\text{bat,R,n}}$)
3. Resultant tangential component of the bat's velocity vector ($v_{\text{bat,R,t}}$)
4. Resultant velocity vector of ball ($v_{\text{ball,R}}$). This is called the ''launch velocity''
5. Resultant normal component of the ball's velocity vector ($v_{\text{ball,R,n}}$)
6. Resultant tangential component of the ball's velocity vector ($v_{\text{ball,R,t}}$)
7. Vertical angle (φ) between the line of centers and $v_{\text{ball,R}}$
8. Vertical angle (λ) between the horizontal plane ($z = 0$) and $v_{\text{ball,R}}$. This is called the ''launch angle''

We also calculate the vertical launch spin rate ($\omega_{\text{ball,R}}$) and specify air density (ρ).

The model is set in the x–z plane of a coordinate system with origin at the contact point, 3 ft in front of the vertex of home plate and at a height of 3 ft. The positive z-axis points upward, positive x-axis points toward the pitcher, and positive y-axis points out of the plane [22]. In Figure 16.14, the x–z plane is reoriented so the x' axis lies along the bat–ball line of centers and the z' axis is tangential to the bat–ball contact point. Angular velocity is positive for pitch topspin and for batted-ball backspin. D is positive if the bat undercuts the ball.

The pitch does not fly horizontally. It is dropping downward at an angle between 4° and 12°, depending on the speed and type of pitch. The angle of descent (γ) of an average fastball is about 10° [10,20]. Batters generally uppercut the ball (ψ) with a 5° to 10° upward angle, which means the ball and bat are actually traveling in opposite directions, as shown in Figure 16.14. For Tables 16.8 through 16.10 we set $\gamma = \psi = 10°$.

16.3.1 Ball's Launch Velocity, Angle, and Spin Rate

The ball's launch parameters are calculated by decomposing the initial velocities of the bat and ball into their normal and tangential components at the point of contact. These velocities are used with the principles of conservation of momentum and conservation of energy to yield resultant normal and tangential velocities for the ball, which are then used to calculate the launch velocity of the ball and the angles φ and λ. The batted-ball

TABLE 16.8

Launch Parameters and Contact Offset for Various Bat–Ball Offsets

Bat–Ball Offset (in.)	Launch Velocity (mph)	Launch Angle (°)	Backspin Rate (rpm)	Contact Offset (in.)
1.50	82	58	4924	0.73
1.25	85	48	3991	0.61
1.00	88	39	3059	0.49
0.75	90	31	2127	0.37
0.50	91	23	1195	0.24
0.25	92	15	263	0.12
0.00	93	8	−669	0

angular velocity is calculated from the normal and tangential linear velocities of the ball and bat and the ball's initial angular velocity.

The collision of two partially elastic bodies with friction is described by numerous authors [48]. The first step in building the model is to calculate θ and, from this, the normal and tangential components of the initial velocity vectors. The vertical angle of the line of centers, $\theta = \psi + \sin^{-1}(D/(r_{\text{bat}} + r_{\text{ball}}))$. The initial velocity components of the ball are $v_{\text{ball,0,n}} = v_{\text{ball,0}} \cos\theta$ and $v_{\text{ball,0,t}} = v_{\text{ball,0}} \sin\theta$. The initial velocity components of the bat are $v_{\text{bat,0,n}} = v_{\text{bat,0}} \cos\theta$ and $v_{\text{bat,0,t}} = v_{\text{bat,0}} \sin\theta$.

The resultant normal velocity of the ball [10] is

$$v_{\text{ball,R,n}} = v_{\text{ball,0,n}} - (1 + CoR)[(m_{\text{bat}}v_{\text{ball,0,n}} - m_{\text{bat}}v_{\text{bat,0,n}})/(m_{\text{ball}} + m_{\text{bat}})] \tag{16.12}$$

Calculation of relative tangential velocity, resultant angular velocity of the ball, and final launch angle, λ, is described by Refs. [22,49]. In these models,

TABLE 16.9

Range and Hang Time for the Launch Parameters of Table 16.8

Bat–Ball Offset (in.)	Launch Velocity (mph)	Launch Angle (°)	Backspin Rate (rpm)	Range (ft)	Hang Time (s)
1.50	82	58	4924	129	6.4
1.25	85	48	3991	236	6.7
1.00	88	39	3059	306	6.1
0.75	90	31	2127	321	5.0
0.50	91	23	1195	285	3.6
0.25	92	15	263	213	2.2
0.00	93	8	−669	122	1.1

TABLE 16.10
Launch Parameters, Range, Hang Time, and the Probability of Batter's Success for Nonnegative Offsets

Bat–Ball Offset (in.)	Launch Velocity (mph)	Launch Angle (°)	Backspin Rate (rpm)	Range (ft)	Hang Time (s)	Probability of Success
1.50	82	58	4924	129	6.4	0.00
1.25	85	48	3991	236	6.7	0.00
1.00	88	39	3059	306	6.1	0.00
0.75	90	31	2127	321	5.0	0.00
0.50	91	23	1195	285	3.6	0.09
0.25	92	15	263	213	2.2	1.00
0.00	93	8	−669	122	1.1	0.63

friction acts in the direction opposite to the slip of the ball. If friction is large enough, it halts the relative tangential velocity (the combined velocities of bat and ball surfaces relative to the contact point). When this occurs, slippage ceases, the ball sticks to the bat, and the ball begins to roll, contributing to the launch angular velocity. These models account for bat recoil and assume conservation of linear and angular momentum for tangential ball and bat motions. Both models ignore deformation of the ball during collision (they assume it remains a perfect sphere).

The launch velocity, launch angle, and backspin rate for various bat–ball offsets are shown in Table 16.8 and Figure 16.15.

Figure 16.15 indicates the launch angle and the center of the ball's area of the contact with the bat. The distance of this contact point from the center axis of the bat can be derived from Figure 16.12. $\sin\theta = \frac{\text{bat-ball offset}}{r_{\text{ball}}+r_{\text{bat}}} = \frac{\text{contact offset}}{r_{\text{bat}}}$ which gives

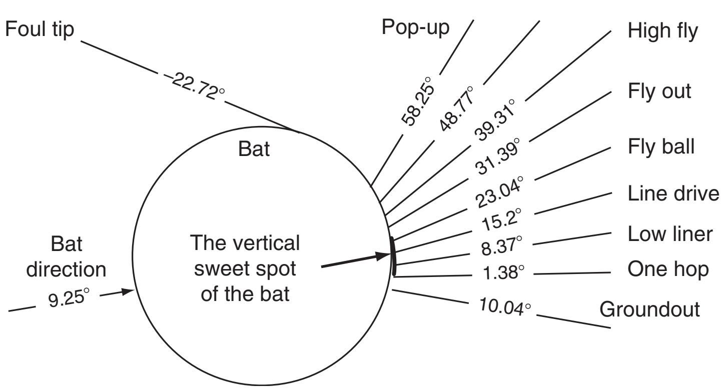

FIGURE 16.15
Common outcomes for some particular launch angles and bat–ball offsets from Table 16.8. The indicated vertical sweet spot of the bat is about one-third of an inch (8 mm) high. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

$$\text{Contact offset} = \frac{\text{bat–ball offset} \times r_{\text{bat}}}{r_{\text{ball}} + r_{\text{bat}}} \tag{16.13}$$

This distance is inserted as an additional column in Table 16.8.

16.3.2 Range, Hang Time, and Ground Time

The launch velocity, launch angle, and spin rate are the input data into the equations of Ref. [10] to calculate the batted ball's range and hang time. The vertical distance traveled by the batted ball (without regard to lift or drag) is $z = v_{z0}\, t - 0.5\, gt^2$, where v_{z0} is the vertical velocity of the ball, t is the hang time, and g is the acceleration rate of gravity at the surface of the Earth (32.17 ft/s^2, 9.8 m/s^2). The horizontal distance traveled (again ignoring lift and drag) is $x = v_{x0}\, t$, where v_{x0} is the horizontal velocity component. However, the rotation of the ball creates a Magnus force acting vertically perpendicular to the trajectory. This force tends to lift the ball (if backspin) or depress the ball (if topspin). It is calculated as $F_{\text{lift}} = 0.5\, \rho \pi r_{\text{ball}}^3\, \omega v_{\text{ball}} \sin \text{VaSa}$, where ρ is the air density. Friction of the ball passing through the air is a drag force acting directly counter to the trajectory. This force is calculated as $F_{\text{drag}} = 0.5\; \rho \pi r_{\text{ball}}^2\; C_{\text{d}}\; v_{\text{ball}}^2$. In our model, the drag and lift coefficients are constants. Table 16.8 shows the ranges and hang times that result from various offsets.

Ground time is not calculated for this chapter. It will be modeled by using the launch angle to find the angle of incidence on the first bounce. The incidental horizontal and vertical velocity components and launch spin rate will then be used to generate the bounce velocity, angle, and spin rate. An aerodynamics model will be used to find the flight characteristics between bounces, including the incidental angle on the subsequent bounce. Note that here CoR and μ will have values different from those for the bat–ball collision. As ω usually represents topspin on ground balls, angular velocity contributes to linear horizontal velocity and vice versa. If μ is large enough to overcome the combined angular and horizontal velocities, slippage stops and rolling begins.

16.3.3 Batting Success Probability Function

The characteristics of a batted ball can be associated with probability of success through a step function based on the potential of defensive players to prevent a base hit. Four kinds of batted-ball behavior are represented in the model:

1. Fly balls (range $>$ 130 ft (40 m), hang time $>$ 2 s)
2. Pop-ups (range $<=$ 130 ft (40 m), hang time $>$ 2 s)
3. Line drives (range $>=$ 115 ft (35 m), hang time $<=$ 2 s)
4. Grounders (range $<$ 115 ft (35 m), hang time $<=$ 1 s)

Our model incorporates several simplifying assumptions. Either an infielder or outfielder might catch a fly ball, depending on range and hang time. All pop-ups are caught by an infielder or a catcher so the probability of success is zero. Only infielders catch line drives and grounders.

Each batted ball is associated with defensive coverage formulated as a function of time. A defensive player prevents a base hit if he or she can reach the ball during hang time or ground time. To determine coverage, we positioned outfielders and infielders on two arcs—the outfield arc with a radius of 300 ft (91 m) and the infield arc with a radius of 115 ft (35 m). The outfield arc is divided into thirds and the infield arc into quarters, with a player positioned at the center of each arc segment. For example, the outfield arc has a length of 471.3 ft (300×1.571); thus, it is divided into three segments each of which is 157.1 ft long. The right fielder, then, is positioned 300 ft from home, 78.55 ft from the right field foul line, and 157.1 ft from the center fielder. The batted ball's range (from the range column of Table 16.10) yields a ''range arc'' with length equal to 1.571 times range (angle in radians times radius).

On fly balls, each player's position is the center of an ellipse representing defensive coverage by the player (a fly ball is illustrated in Figure 16.16). Hang time determines the dimensions of the ellipse for a specific batted ball. Probability of a base hit is the proportion of the range arc that is not overlapped by ellipses.

In Figure 16.16, the outfielders are positioned on the outfield arc. The dashed line shows the range arc for a low fly ball that is in the air for 3 s and travels 250 ft. Three-fourth of this range arc is overlapped by the 3 s fielder ellipses. Therefore, the probability of success is 0.25.

If a line drive or grounder passes the infield arc without encountering an infielder, it is considered a base hit. Therefore, only infielders' lateral

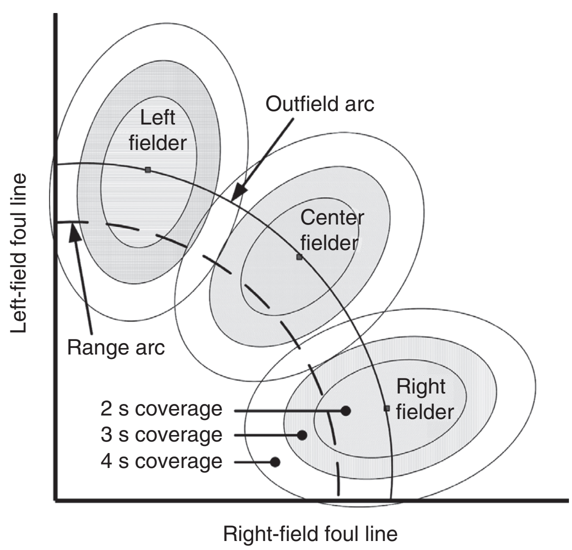

FIGURE 16.16
Range arc, outfield arc, and defensive coverage of each outfielder for batted balls that would be in the air for 2, 3, and 4 s. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2004.)

movements provide coverage. Success is the proportion of range arc (on line drives) or infield arc (on grounders) not covered by infielders. In this model, batters do not beat out infield hits and pitchers do not catch line drives or grounders.

The model assumes the speed of outfielders and infielders is 23 ft/s (7 m/s). Outfielders' reaction delays are 8 ft (2.4 m) (0.35 s) forward, 12 ft (3.7 m) (0.52 s) sideward, and 15 ft (4.6 m) (0.65 s) backward. Infielders' reaction times are 12 ft sideward and 15 ft backward. These values were selected as ''reasonable'' and are not based on empirical data.

16.3.4 Example of Varying Offsets

For an example of collision evaluation, the model is solved at offset increments of 0.25 in. upward from zero offset. Pitch backspin is −1800 rpm and pitch speed is 85 mph (38 m/s). Contact occurs at the bat's area of maximum horizontal sweetness and the speed of the bat's contact point is 60 mph (27 m/s) the average value for the San Francisco Giants [43]. These speeds produce a CoR of 0.54. We measured the coefficient of friction, μ, to be 0.5 (see also Ref. [22]). The angles γ and ψ of ball and bat are both 10°. Other test values are $r_{\text{bat}} = 1.375$ in., $r_{\text{ball}} = 1.452$ in., $m_{\text{bat}} = 32.0$ oz (effective bat mass = 20.0 oz [33]), $m_{\text{ball}} = 5.125$ oz, and ρ at standard sea level conditions. These numbers are given in Tables 16.2 and 16.11. Ranges and hang times were found using a Pascal aerodynamics program. Launch

TABLE 16.11

Parameter Values Used to Compute the Vertical Size of the Sweet Spot SI Units

	Major League Baseball	**Little League**	**NCAA Softball**
Bat type	Wooden C243	Aluminum	Aluminum
Ball type	Baseball	Baseball	Softball
Pitch speed (m/s)	38	22	29
Speed of sweet spot (m/s)	27	20	22
CoR	0.54	0.53	0.52
Typical bat mass (kg)	0.9	0.6	0.7
Ball mass (kg)	0.145	0.145	0.191
Maximum bat radius (m)	0.035	0.029	0.029
Ball radius (m)	0.037	0.037	0.048
Distance from front of rubber to tip of plate (m)	18.4	14.0	13.1
Pitcher's release point: distance from tip of plate and height	17 m out 2 m up	13 m out 1.5 m up	12 m out 0.8 m up
Bat–ball collision point: distance from tip of plate and height	1 m out 1 m up	1 m out 1 m up	1 m out 1 m up
Backspin of batted ball (rad/s)	100–500	100–500	100–500
Desired ground contact point: distance from the plate (m)	37–73	24–43	24–46
Air density, ρ (kg/m^3)	1.04	1.04	1.04

Note: Air density is inversely related to temperature, altitude, and humidity.

values were computed using spreadsheets developed by A. Nathan (personal communication).

16.3.5 Results

Test results are given in Table 16.10. As hang time increases, probability of success decreases rapidly. Pop-ups are produced by offsets greater than 1.5 in. (3.8 cm). These are assigned a success probability of zero. Note the model assumes no outfield barriers. In most major league stadiums, long fly balls have a chance of clearing the wall (the average distances are 330 ft (100 m) down the foul lines and 400 ft (122 m) in center field). Thus, the model underestimates success for any range with a chance to be a home run.

The example shows how the model might be used to analyze collision parameters (e.g., offsets, bat velocity) or bat properties (e.g., bat radius). Relating initial conditions to sweetness provides a valuable criterion for these analyses.

16.3.6 Discussion

From this collision model, we get the launch velocity, the launch angle, and the backspin rate. We put these into our simulation for the batted ball that uses the following equations from Ref. [10, p. 80]:

$$F_{\text{drag}} = 0.25\,\rho\pi r_{\text{ball}}^2 v_{\text{ball-after}}^2 \tag{16.14}$$

$$F_{\text{Magnus}} = 0.5\,\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball-after}} \tag{16.15}$$

where

ρ is air density
$v_{\text{ball-after}}$ is the ball speed after its collision with the bat
ω is the rotation rate
r_{ball} is the radius of the ball

Values for these parameters are provided in Tables 16.1 and 16.11.

Some physicists (see Equation 1 in Ref. [25]) model the Magnus force with $F_L = \frac{1}{2} C_L \rho A v^2$, where A is the cross-sectional area of the ball and C_L is not a constant, but rather it is a nonlinear parameter that depends on the Reynolds number, the spin rate, the ball velocity, and, perhaps, C_D. However, we prefer the simpler formulation of Equation 16.15.

To show how Equations 16.14 and 16.15 work, let us now present a simple numerical example. Assume a 95 mph (42.5 m/s) fastball has 20 rps of pure backspin. Near the beginning of the pitch, the Magnus force will be straight up in the air, i.e., pure lift. Using English units and Table 16.1, we get

$$\begin{aligned} F_{\text{drag}} &= 0.25\,\rho\pi r_{\text{ball}}^2 v_{\text{ball}}^2 \\ &= (0.25)(0.0023)(3.14)(0.12)^2(139)^2 = 0.5\,\text{lb} \end{aligned}$$

and

$$F_{\text{Magnus}} = 0.5\,\rho\pi r_{\text{ball}}^{3}\omega v_{\text{ball-after}}$$
$$= (0.5)(0.0023)(3.14)(0.12)^{3}(126)(139) = 0.11\,\text{lb}$$

which is about one-third the force of gravity given in Table 16.1. This is consistent with Tables 16.3 and 16.4.

Using SI units and Table 16.11, we get

$$F_{\text{drag}} = 0.25\,\rho\pi r_{\text{ball}}^{2} v_{\text{ball}}^{2}$$
$$= (0.25)(1.2)(3.14)(0.037)^{2}(42.5)^{2} = 2.3\,\text{n}$$

and

$$F_{\text{Magnus}} = 0.5\,\rho\pi r_{\text{ball}}^{3}\omega v_{\text{ball-after}}$$
$$= (0.5)(1.2)(3.14)(0.037)^{3}(126)(42.5) = 0.51\,\text{n}$$

which is about one-third the force of gravity, which is

$$F_{\text{gravity}} = mg = 0.145 \times 9.8 = 1.42\,\text{n} \tag{16.16}$$

This simulator allows us to calculate the trajectory of the batted ball. From the ball's trajectory we can compute where it will first hit the ground. Assume that the batter wants to hit a line drive that first hits the ground between 120 and 240 ft (37 to 73 m) from the plate. (The performance criterion is to maximize the probability that the batted ball will be a line drive that first hits the ground 120 to 240 ft from the plate.) From our simulations, the vertical offset between the ball and the bat should be between 0.15 and 0.45 in. (0.38 to 1.1 cm). Therefore, the vertical size of the sweet spot of the bat is one-third of an inch (8 mm). For the Little League the vertical size of the sweet spot is about the same. However, because the softball is bigger, for NCAA softball the vertical size of the sweet spot is a little less than half an inch.

This discussion is suggesting another performance criterion: efficiency. The batter wants to swing the bat so that as much energy as possible is transferred from the bat to the ball in a particular direction, namely 5° to 10° upward. Momentum in a perpendicular direction is not helpful (pop-ups and grounders). This performance criterion wants the batted-ball direction to be the same as the bat's direction before the collision, i.e., it wants a 5° to 10° uppercut and zero offset. A lot of previously used performance criteria were appropriate for home runs. This new performance criterion is designed for line drive singles or doubles.

At this point it is appropriate to caution young players; we are not advising that they ignore their coaches' advice to ''swing level.'' Coaches and parents have difficulty differentiating between level horizontal swings and those with a 5° to 10° upward angle. The coach's admonition means do not swing with a 30° upward angle, because you do not want to launch the ball at 30°. In this context, swing level means swing with a 5° to 10° upward angle.

16.4 Swing of the Bat

Williams and Underwood [41] said that hitting a baseball is the hardest act in all of sports. This act is easier if the right bat is used, but it is difficult to determine the right bat for each individual. Therefore, we developed the Bat Chooser* to measure the swings of an individual, make a model for that person, and compute his or her Ideal Bat Weight [4,43]. The Bat Chooser uses individual swing speeds, CoR data, and the laws of conservation of momentum, and then it computes the ideal bat weight for each individual, trading off maximum batted-ball speed with accuracy. However, with the advent of lightweight aluminum bats, it is now possible for bat manufacturers to vary not only the weight but also the weight distribution. They can start with a lightweight aluminum shell, and add a weight inside the barrel to bring the bat up to its specified weight. This internal weight can be placed anywhere inside the barrel. When the weight is placed at the tip of the bat, the bat is said to be ''end loaded.'' So now, there is a need to determine the best weight distribution in general, for certain classes of players and for individual players. These are the topics of this section.

16.4.1 Ideal Bat Weight and the Bat Chooser

Our instrument for measuring bat speeds, the Bat Chooser, has two vertical laser beams, each with an associated light detector. The subjects were positioned so that when they swung the bats, the sweet spot (which we defined to be an area on the bat that is centered 5 in. from the barrel end) of each bat passed through the laser beams. A computer (sampling once every 16 μs) recorded the time between interruptions of the laser beams. Knowing the distance between the laser beams (15 cm, 6 in.) and the time required for the bat to travel that distance, the computer calculated the horizontal speed of the bat's sweet spot for each swing. This is a simple model, because the motion of the bat is very complex, being comprised of a horizontal translation, a rotation about the batter's spine, a rotation about a point between the two hands (which may be moving), and a vertical motion.

In our variable moment of inertia experiments, to be described in the next section, and in our ideal bat weight experiments, each player was positioned so that bat speed was measured at the place where the subject's front foot hit the ground. We believe that this is the place where most players reach maximum bat speed. The batters were told to swing each of six bats as

* Bat Chooser and Ideal Bat Weight are trademarks of Bahill Intelligent Computer Systems.

fast as possible, while still maintaining control. They were told to ''Pretend you are trying to hit a Randy Johnson fastball.'' In a 20 min interval of time, each subject swung each bat through the instrument five times. The order of presentation was randomized. A speech synthesizer announced the selected bat; for example, ''Please swing bat Babe Ruth; that is bat B.'' For each swing, the name of the bat and the speed of the sweet spot were recorded.

To reduce bat swing variability we gave the batters a visual target to swing at. It was a knot on the end of a string hanging from the ceiling. Typically, this knot was 3 ft (1 m) off the floor. The height of this knot was very important for some batters. For one batter, bat speed increased 20% when the knot was lowered 1 ft (0.3 m).

16.4.2 Principles of Physics Applied to Bat Weight Selection

The speed of a baseball after its collision with a bat depends on many factors, not the least of which is the weight of the bat. In this section, we present data to help an individual player to decide if his or her preference is the most effective bat weight. Knowing the ideal bat weight can eliminate time-consuming and possibly misleading experimentation by ball players.

To find the best bat weight we must first examine the conservation of momentum equations for bat–ball collisions.

$$m_{\text{bat}} v_{\text{bat-before}} + m_{\text{ball}} v_{\text{ball-before}} = m_{\text{bat}} v_{\text{bat-after}} + m_{\text{ball}} v_{\text{ball-after}} \tag{16.17}$$

We want to solve for the ball's speed after its collision with the bat, called the ''batted-ball speed,'' but first we should eliminate the bat's speed after the collision, because it is not easily measured. The CoR for a bat–ball collision can be modeled with

$$CoR = -\frac{v_{\text{bat-after}} - v_{\text{ball-after}}}{v_{\text{bat-before}} - v_{\text{ball-before}}} \tag{16.18}$$

The negative signs are there because $v_{\text{ball-before}}$ is in the direction from the pitching rubber to the plate, whereas the other three velocities go from the plate toward the rubber. Therefore, we define $v_{\text{ball-before}}$ to have a negative magnitude.

We can use the equation for the CoR to solve for $v_{\text{ball-after}}$, substitute the result into the equation for the conservation of momentum, and solve for the ball's speed after its collision with the bat. The result is

$$v_{\text{ball-after}} = \frac{-v_{\text{ball-before}}\left[CoR - \dfrac{m_{\text{ball}}}{m_{\text{bat}}}\right] + (1 + CoR)v_{\text{bat-before}}}{1 + \dfrac{m_{\text{ball}}}{m_{\text{bat}}}} \tag{16.19}$$

This means that the ball's speed after the collision will depend on the mass of the ball, the mass of the bat, the CoR, and the precollision speeds of the ball and bat.

16.4.3 Coupling Physics to Physiology

Physiologists have long known that muscle speed decreases with increasing load. This is why bicycles have gears. The rider can keep muscle speed in its optimal range while bicycle speed varies greatly. Therefore, to discover how muscle properties of individual ball players affect their best bat weights, we measured the bat speeds of many batters swinging bats of various weights. We plotted the data of bat speed versus bat weight, and used this to help calculate the best bat weight for each batter.

Over the last half century, physiologists have used three equations to describe the force–velocity relationship of muscles: that for the straight line ($y = Ax + B$), that for the rectangular hyperbola ($(x + A)\ (y + B) = C$), and that for the exponential ($y = Ae^{-Bx} + C$). Each of these equations has been best for some experimenters, under some conditions, with certain muscles, but usually the one for the hyperbola fits the data best. In our experiments, we fit all three and chose the equation that gave the best fit to the data of each subject's 30 swings. For example, for batters where the straight line fit was the best

$$v_{\text{bat-before}} = slope\ m_{\text{bat}} + intercept \tag{16.20}$$

where

slope is the slope of the line
intercept is the *y*-axis intercept.

Now to couple physiology to physics, we substituted this relationship into the previous equation to yield

$$v_{\text{ball-after}} = \frac{-v_{\text{ball-before}}\left[CoR - \dfrac{m_{\text{ball}}}{m_{\text{bat}}}\right] + (1 + CoR)(slope\ m_{\text{bat}} + intercept)}{1 + \dfrac{m_{\text{ball}}}{m_{\text{bat}}}} \tag{16.21}$$

Next, you can either take the derivative with respect to the bat weight, set this equal to zero, and solve for the maximum batted-ball speed bat weight or you can get this result graphically, as suggested in Figure 16.17.

16.4.4 Ideal Bat Weight

The maximum batted-ball speed bat weight is probably not the best bat weight for any player. A lighter bat will give a player better control and more accuracy. Obviously, a trade-off must be made between maximum

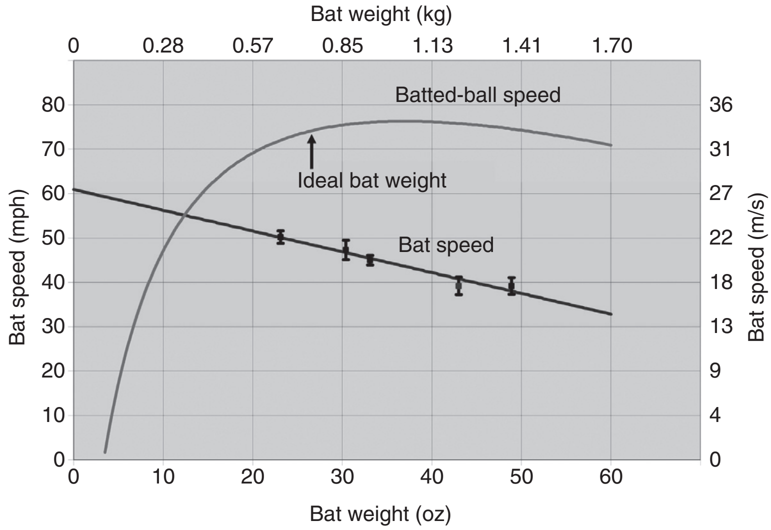

FIGURE 16.17
Bat speed (straight line) and batted-ball speed (curved line) for a typical member of the University of Arizona softball team. Her ideal bat weight is 28 oz. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2007.)

batted-ball speed and controllability. Because the batted-ball speed curve is so flat around the point of the maximum batted-ball speed bat weight, we believe there is little advantage in using a bat as heavy as the maximum batted-ball speed bat weight. Therefore, we have defined the ideal bat weight to be the weight at which the ball speed curve drops 2% below the speed of the maximum batted-ball speed bat weight. We believe this gives a reasonable trade-off between distance and accuracy.* Of course, this is subjective and each player might want to weigh the two factors differently. It does, however, give a quantitative basis for comparison. For the player whose data are shown in Figure 16.17, the ideal bat weight was 28 oz (0.8 kg).

Not only is the ideal bat weight specific for each player, but it also depends on whether the player is swinging right or left handed. We measured two switch-hitters (one professional and one university ball player who later had a long professional career). One player's ideal bat weights were 1 oz (0.03 kg) different and the other's were 5 oz (0.14 kg) different. Switch-hitters were so different when hitting right and left handed that we treated them as different players.

It is difficult for most batters to determine the best bat for themselves. Therefore, we developed a system to measure the swings of an individual,

* We used 1% for major league baseball players and NCAA softball champions.

TABLE 16.12

Rules of Thumb for Recommending Bats

Group	Recommended Bat Weight
Baseball, major league	Height/3 + 7
Baseball, amateur	Height/3 + 6
Softball, fast-pitch	Height/7 + 16
Softball, slow-pitch	Weight/115 + 24
Junior league (13 and 15 years)	Height/3 + 1
Little League (11 and 12 years)	Weight/18 + 16
Little League (9 and 10 years)	Height/3 + 4
Little League (7 and 8 years)	2 × Age + 4

Note: Recommended bat weight is in ounces, age is in years, height is in inches, and body weight is in pounds.

make a model for that person, and recommend a specific bat weight for that person. However, this system is not conveniently available to most people. So we used our database of the 200 people who had been measured with our system and created simple equations that can be used to recommend a bat for an individual using common parameters such as age, height, and weight. These recommendations are given in Table 16.12. These rules of thumb were derived from our 200 subject database, with constraints of commercial availability and integer numbers, from Ref. [28].

16.4.5 Ideal Moment of Inertia

Bahill [2] presented the variable moment of inertia data that his group has gathered over the last two decades. In these studies, the subjects swung bats of the same weight, but different weight distribution (inertia). The bat speeds were measured and recorded. Then the data for each player were fit with a line of the form

$$v_{\text{bat-before}} = slope\ I_{\text{knob}} + intercept \tag{16.22}$$

where

slope is the slope of the line
I_{knob} is the moment of inertia of the bat with respect to the knob
intercept is the y-axis intercept

We model the swing of a bat as a translation and two rotations: one centered in the batter's body and the other between the batter's hands. Next, we compute the batted-ball speed (the speed of the ball after its collision with the bat). We use conservation of linear and angular momentum and the definition of the CoR to get the following equation, which has been previously derived [10,36]:

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$$v_{\text{ball-after}} = \frac{-v_{\text{ball-before}}\left[CoR - \dfrac{m_{\text{ball}}}{m_{\text{bat}}} - \dfrac{m_{\text{ball}}\, d^2_{\text{cm-ss}}}{I_{\text{cm}}}\right] + (1 + CoR)v_{\text{bat-before}}}{1 + \dfrac{m_{\text{ball}}}{m_{\text{bat}}} + \dfrac{m_{\text{ball}}\, d^2_{\text{cm-ss}}}{I_{\text{cm}}}} \tag{16.23}$$

where

CoR is the coefficient of restitution of the bat–ball collision

$d_{\text{cm–ss}}$ is the distance between the center of mass and the sweet spot, which is assumed to be the point of collision

I_{cm} is the moment of inertia about the center of mass.

The term $v_{\text{bat-before}}$ is simply the velocity of the sweet spot. $v_{\text{ball-before}}$ is a negative number, because its direction is the opposite of $v_{\text{ball-after}}$.

The subjects swung bats composed of wooden bat handles with ¼ inch threaded rods attached to the end and brass disks fixed at various points on the rods. These bats had similar lengths and masses, but a wide range for moments of inertia. The moment of inertia of a bat is given with

$$I_{\text{knob}} = I_{\text{handle}} + m_{\text{disk}} d^2_{\text{knob-disk}} \tag{16.24}$$

where

I_{knob} is the inertia of the total bat with respect to the knob

I_{handle} is the inertia of the handle part of the bat with respect to the knob

m_{disk} is the mass of the disk on the end of the rod

$d_{\text{knob–disk}}$ is the distance from the knob to the disk

After a little bit of algebra, Bahill [2] derived the following equation for the batted-ball speed:

$$v_{\text{ball-after}} = v_{\text{ball-before}} + \frac{(1+CoR)[slope(I_{\text{handle}} + m_{\text{disk}} d^2_{\text{knob-disk}}) + intercept - v_{\text{ball-before}}]}{1 + \dfrac{m_{\text{ball}}}{m_{\text{bat}}} + \dfrac{m_{\text{ball}}\left(d_{\text{k-ss}} - \dfrac{m_{\text{disk}} d_{\text{knob-disk}}}{m_{\text{handle}} + m_{\text{disk}}} - \dfrac{m_{\text{handle}} d_{\text{knob-cm(handle)}}}{m_{\text{handle}} + m_{\text{disk}}}\right)^2}{I_{\text{handle}} + d^2_{\text{knob-disk}}\left(m_{\text{disk}} - \left[\dfrac{m_{\text{disk}} d_{\text{knob-disk}}}{m_{\text{handle}} + m_{\text{disk}}}\right]^2 m_{\text{bat}}\right) - \dfrac{2 m_{\text{disk}} d_{\text{knob-disk}}}{m_{\text{handle}} + m_{\text{disk}}} \dfrac{m_{\text{handle}} d_{\text{knob-cm(handle)}}}{m_{\text{handle}} + m_{\text{disk}}} - \left(\dfrac{m_{\text{handle}} d_{\text{knob-cm(handle)}}}{m_{\text{handle}} + m_{\text{disk}}}\right)^2 m_{\text{bat}}}}$$

This equation is plotted in Figure 16.18 for a typical subject.

All of the batters in this study would profit (meaning would have higher batted-ball speeds) from using end-loaded bats.

At this point, it may be useful to reiterate that an end-loaded bat is not a normal bat with a weight attached to its end. Adding a weight to the end of a normal bat would increase both the weight and the moment of inertia. This is unlikely to help anyone. In the design and manufacture of an

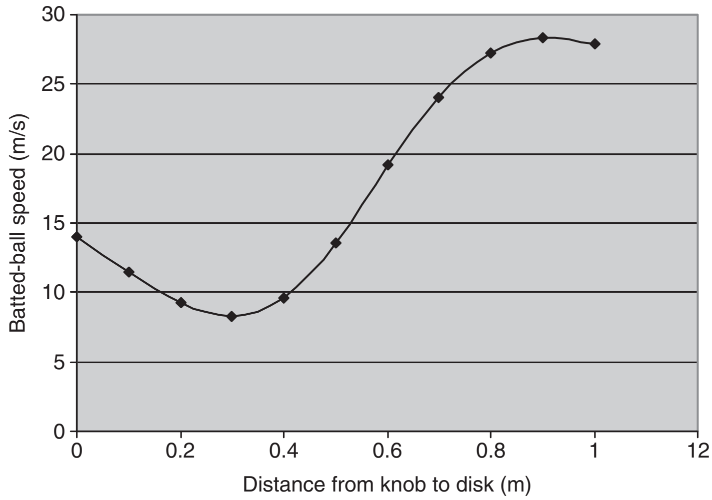

FIGURE 16.18
Batted-ball speed as a function of $d_{\text{knob–disk}}$ for one batter showing an optimal value at 0.9. (From Bahill, A.T., http://www.sie.arizona.edu/sysengr/slides. With permission. Copyright 2003.)

end-loaded bat, the weight is distributed so that the bat has a normal weight, but a larger than normal moment of inertia.

16.5 Summary

This chapter presented the right-hand rules that can be used to show the direction of spin-induced deflection for a spinning ball in any sport. They were summarized with the acronym SaD Sid. Then, we discussed the sweet spot of the bat. Nine different definitions were given for the horizontal sweet spot of a bat: most of them were in an area 5 to 7 in. (13 to 18 cm) from the end of the barrel. Next, this chapter presented a new model for bat–ball collisions and used it along with a new performance criterion, namely the probability of getting a hit. Previous models were designed for analyzing home runs, which constitute less than 4% of the batted balls in play. This new model was used to describe the vertical gradients of the sweet spot of the bat. The vertical size of the sweet spot is one-third of an inch (8 mm). Then the chapter showed that there is an ideal bat weight for each batter. A simple table gave rules of thumb for recommending bat weights. Finally, this chapter gave a recommendation that all batters would profit from using end-loaded bats. For nonmathematical aspects of baseball see Baldwin's autobiography, Snake Jazz [50].

List of Variables

CoP	Center of percussion of a bat
CoR	Coefficient of restitution of a bat–ball collision
CollisionSpeed	Sum of pitch speed and speed of the bat at the collision point
C_d	Coefficient of drag
d_{cm-cop}	Distance from the center of mass to the center of percussion
d_{cm-ss}	Distance from the center of mass to the sweet spot
$d_{knob-cm}$	Distance from the center of the knob to the center of mass
$d_{pivot-ss}$	Distance from the pivot point to the sweet spot
$d_{pivot-cm}$	Distance from the pivot point to the center of mass
$d_{pivot-cop}$	Distance from the pivot point to the center of percussion
g	Earth's gravitational constant
I_{cm}	Moment of inertia of the bat with respect to the center of mass
I_{knob}	Moment of inertia of the bat with respect to the knob
I_{pivot}	Moment of inertia of the bat with respect to the pivot point
m_{ball}	Mass of the ball
m_{bat}	Mass of the bat
r_{ball}	Radius of the ball
r_{bat}	Radius of the bat
$v_{ball\text{-}after}$	Speed of the ball after the bat–ball collision
$v_{ball\text{-}before}$	Speed of the ball before the bat–ball collision
$v_{bat\text{-}after}$	Speed of the bat after the bat–ball collision
$v_{bat\text{-}before}$	Speed of the bat before the bat–ball collision
ω	Ball rotation rate

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