Effects of Altitude and Atmospheric Conditions on the Flight of a Baseball

A. Terry Bahill [1, +], David G. Baldwin [2] and John S. Ramberg [1]

[1] Systems and Industrial Engineering, University of Arizona, Tucson, AZ 85721-0020

[2] Chicago White Sox Baseball Team, retired

Abstract. Altitude and weather affect air density, which in turn affects how far a batted baseball or softball travels. This paper shows that air density is inversely related to altitude, temperature and humidity, and is directly related to barometric pressure. Regression analysis is used to show the relative importance of each of the four factors (altitude, temperature, humidity, and barometric pressure) and to look for interactions between them. As shown by this model, on a typical July afternoon in a major league baseball stadium, altitude is easily the most important factor, explaining 80% of the variability. This is followed by temperature (13%), barometric pressure (4%) and relative humidity (3%). A simple linear algebraic equation presented in this paper predicts air density well. The model shows how the batted ball's range depends on both the drag force and the Magnus force (the force due to a spinning object moving in an airflow) and considers the relative importance of the drag and Magnus forces.

Keywords: baseball, softball, home run, altitude, temperature, barometric pressure, relative humidity, air density, sensitivity analysis, equations, Physics, slider, spin of a ball, Magnus force, drag force,

1. Movement of the Pitch

Baseball batters say that the pitch hops, drops, curves, breaks, rises, sails or tails away. Baseball pitchers say that they throw fastballs, screwballs, curveballs, drop curves, flat curves, knuckle curveballs, sliders, change ups, palm balls, split fingered fastballs, splitters, forkballs, sinkers, cutters, two-seam fastballs and four-seam fastballs. This sounds like a lot of variation. However, no matter how the pitcher grips or throws the ball, once it is in the air its motion depends only on gravity, its velocity and its spin. (This statement is true even for the knuckleball, because it is the shifting position of the seams during its slow spin en route to the plate that gives the ball its erratic behavior.) In engineering notation, these pitch characteristics are described respectively by a *linear velocity vector* and an *angular velocity vector*, each with magnitude and direction. The magnitude of the linear velocity vector is called *pitch speed* and the magnitude of the angular velocity vector is called the *spin rate*. These vectors produce a force acting on the ball that causes a deflection of the ball's trajectory. The first sections of this paper are based on [Bahill and Karnavas, 1991; Bahill and Baldwin, 2003; Bahill, 2004; Bahill, Botta, and Daniels, 2006; Bahill and Baldwin, 2007; Bahill and Baldwin, 2008].

Isaac Newton [1671] noted that spinning tennis balls experienced a lateral deflection mutually perpendicular to the direction of flight and to the direction of spin. Later, Benjamin Robins [1742] bent the barrel of a musket to produce spinning musket balls and also noted that the spinning balls experienced a lateral deflection perpendicular to the direction of flight and to the direction of spin. In 1853, Gustav Magnus (see [Briggs, 1959; Barkla and Auchterlonie, 1971] studied spinning artillery shells fired from rifled artillery pieces and found that the range depended on crosswinds. A crosswind from the right lifted the shell and gave it a longer range: a crosswind from the left made it drop short. In 1902 the Polish born Martin Kutta and independently in 1906 Nikolai Joukowski studied cylinders spinning in an airflow. They were the first to model this force with an equation. Although these four experiments sound quite different (and they did not

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+ Corresponding author. A. Terry Bahill, Systems and Industrial Engineering, University of Arizona, Tucson, AZ 85721-0020, (520) 621-6561, terry@sie.arizona.edu.
David G. Baldwin, P.O. Box 190, Yachats, OR 97498, (541) 547-4713, snakejazz_37@brainpip.com.
John S. Ramberg, Professor Emeritus, 114 Capricho Circle, Pagosa Springs, CO 81147, (970) 731-2413, jsramberg@msn.com.

know about each other's work), they were all investigating the same underlying force. This force, commonly called the Magnus force, operates when a spinning object (like a baseball) moves through a fluid (like air) which results in it being pushed sideways. Two models explain the basis of this Magnus force: one is based on conservation of momentum and the other is based on Bernoulli's principle [Bahill and Karnavas, 1993; Watts and Bahill, 2000; NASA, 2008].

Figures 1 and 2 show the effects of spin on the pitch. During the pitch of a major league baseball, the ball falls about three feet due to gravity ($d = \frac{1}{2}at^2$). However, the fastball has backspin that opposes gravity and the curve ball has top spin that aids the fall due to gravity.
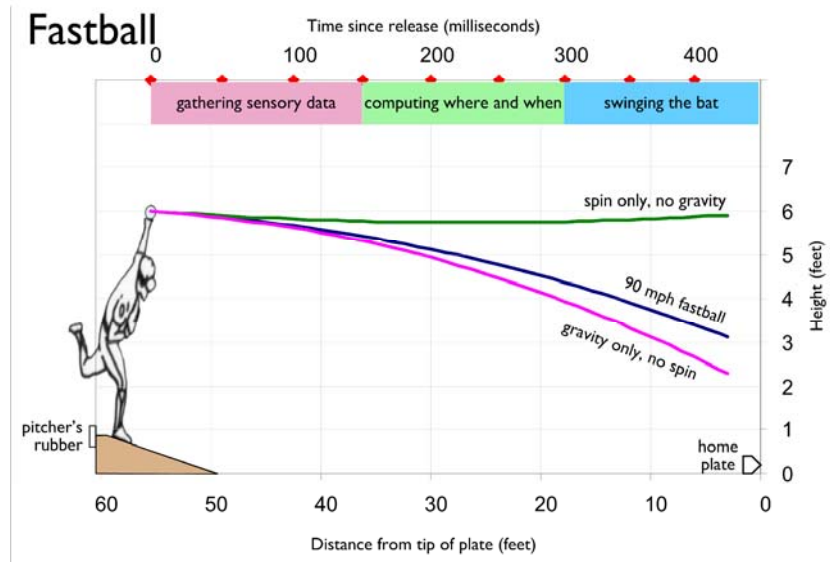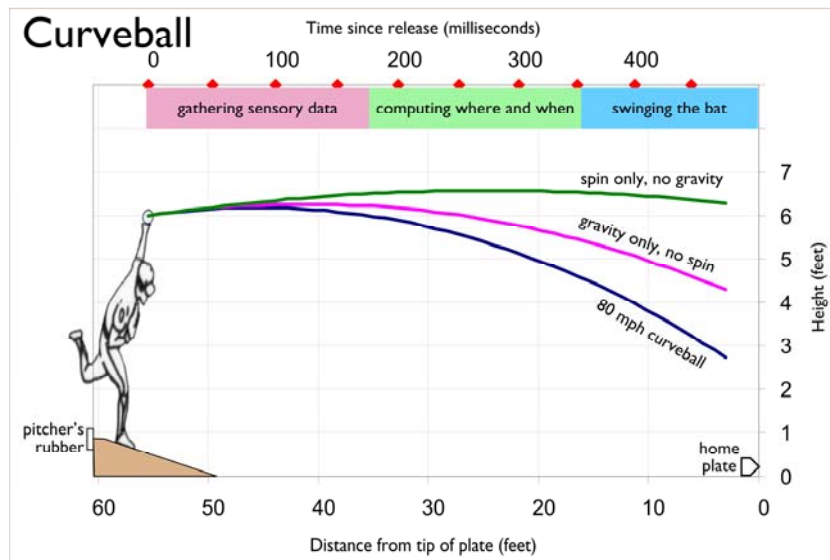

Fig. 1. A 90 mph (40 m/s) overhand fastball launched one-degree downward with 1200 rpm of backspin. Copyright ©,

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Fig. 2. An 80 mph (36 m/s) overhand curveball launched two-degrees upward with 2000 rpm of topspin. Copyright ©, 2008, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

In the simulations of Figures 1 and 2, the pitcher releases the ball five feet (1.5 m) in front of the rubber at a height of six feet (1.8 m). The batter hits the ball 1.5 feet (0.5 m) in front of the plate. These figures also show what the batter is doing during the pitch. During the first third of the pitch, he is gathering sensory information (mostly with his eyes) about the speed and spin of the pitch. During the middle third of the pitch, he is computing where and when the ball will cross the plate. During the last 150 msec, he is swinging the bat and can do little to alter its trajectory.

We will now apply the right-hand rules to the linear velocity vector and the angular velocity vector in order to describe the *direction* of the spin-induced deflection of the a spinning ball in flight.

2. Right-hand Rules Applied to a Spinning Ball in Flight

First we use the *angular right-hand rule* to find the direction of the spin axis. As shown in Figure 3, if you curl the fingers of your right hand in the direction of spin, your extended thumb will point in the direction of the spin axis.

Fig. 3. The angular right-hand rule. When the fingers are curled in the direction of rotation, the thumb points in the direction of the spin axis. Photograph by Zach Bahill. Copyright ©, 2004, Bahill, used with permission from http://www.sie.arizona.edu/sysengr/slides/.

Fig. 4. The coordinate right-hand rule. If the thumb points in the direction of the spin axis and the index finger points in the direction of forward motion, then the middle finger will point in the direction of the spin-induced deflection. Photograph by Zach Bahill. Copyright ©, 2004, Bahill, used with permission from http://www.sie.arizona.edu/sysengr/slides/

Next we use the *coordinate right-hand rule* to determine the direction of the spin-induced deflection force. Point the thumb of your right hand in the direction of the spin axis (as determined from the angular right-hand rule), and point your index finger in the direction of forward motion (Figure 4). Bend your middle finger so that it is perpendicular to your index finger. Your middle finger will be pointing in the direction of the spin-induced deflection (of course, the ball also drops due to gravity). The spin-induced deflection force will be in a direction represented by the cross product of the angular velocity vector (the spin axis) and the linear velocity vector of the ball: Angular velocity × Linear velocity = Spin-induced deflection force. Or mnemonically, Spin axis × Direction = Spin-induced deflection (SaD Sid). This acronym only gives the direction of spin-induced deflection. The equations yielding the magnitude of the spin-induced deflection force are discussed in section 4.

3. Direction of Forces Acting on Specific Pitches

Figures 5 and 6 show the directions of spin (circular red arrows) and spin axes (straight black arrows) of some common pitches from the perspective of the pitcher (Figure 5 represents a right-hander's view and Figure 6 a left-hander's view). We will now consider the direction of the spin-induced deflection of each of these pitches.
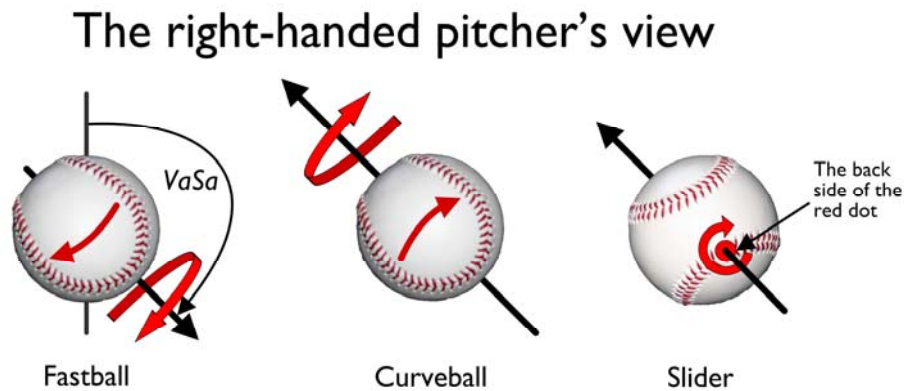
The right-handed pitcher's view

Fig. 5. The direction of spin (circular red arrows) and the spin axes (straight black arrows) of a three-quarter arm fastball, an overhand curveball and a slider, all from the perspective of a right-handed pitcher, meaning the ball is moving into the page. *VaSa* is the angle between the Vertical axis and the Spin axis (*VaSa*). The spin axes could be labelled spin vectors, because they suggest both magnitude and direction. Copyright ©, 2005, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.
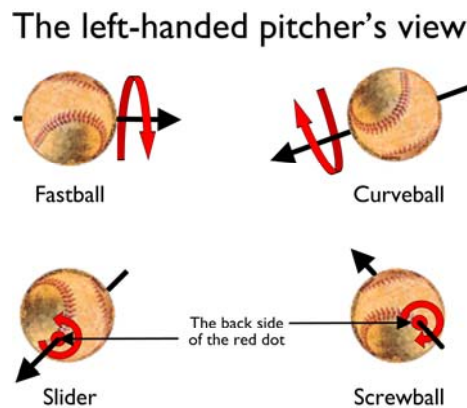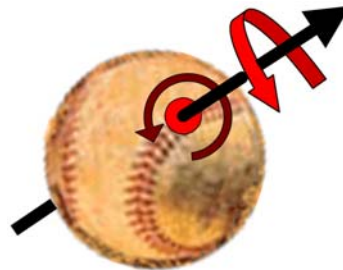
The left-handed pitcher's view

Fig. 6. The direction of spin (circular arrows) and the spin axes (straight arrows) of an *overhand* fastball, an overhand curveball, a slider and a screwball from the perspective of a *left*-handed pitcher. The ball would be moving into the page. Copyright ©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

The spin on the ball is produced by the grip of the fingers and the motion of the pitcher's arm and wrist. When a layperson throws a ball, the fingers are the last part of the hand to touch the ball. If the ball is thrown with an overhand motion, the fingers touch the ball on the bottom and thus impart backspin to the ball. The overhand fastball shown in Figure 6 has predominantly backspin, which gives it lift, thereby decreasing its fall due to gravity as shown in Figure 1. However, most pitchers throw the fastball with a three-quarter arm delivery, which means the arm does not come straight over-the-top, but rather it is in between over-the-top and sidearm. This delivery rotates the spin axis from the horizontal as shown for the fastball in Figure 5. This rotation of the axis reduces the lift and also introduces lateral deflection, to the right for a right-handed pitcher.

The curveball can also be thrown with an overhand delivery, but this time the pitcher rolls his wrist and causes the fingers to sweep in front of the ball. This produces a spin axis as shown for the overhand curveball of Figure 5. This pitch will curve at an angle from upper right to lower left as seen by a right-handed pitcher. Thus, the ball curves diagonally. The advantage of the drop in a pitch is that the sweet area of the bat is about two inches long (5 cm) [Bahill, 2004] but only one-third of an inch (8 mm) high [Bahill and Baldwin, 2003; Baldwin and Bahill, 2004]. Thus, when the bat is swung in a horizontal plane, a vertical drop is more effective than a horizontal curve at taking the ball away from the bat's sweet area.

The slider is an enigmatic pitch. It is thrown somewhat like a football. Unlike the fastball and curveball, the spin axis of the slider is not perpendicular to the direction of forward motion. As the angle between the spin axis and the direction of motion decreases, the magnitude of deflection decreases, but the direction of deflection remains the same. If the spin axis is coincident with the direction of motion, as for the backup

slider [Bahill and Baldwin, 2007, footnote 3], the ball spins like a bullet and experiences no deflection. Therefore, a right-handed pitcher usually throws the slider so that he sees the axis of rotation pointed up and to the left. This causes the ball to drop and curve from the right to the left. Rotation about this axis allows some batters to see a red dot at the spin axis on the upper-right-side of the ball (See Figure 7). Baldwin, Bahill and Nathan [2007] and Bahill, Baldwin and Venkateswaran [2005] show pictures of this spinning red dot. Videos of this spinning red dot are on Bahill's web site (www.sie.arizona.edu/sysengr/baseball/). Seeing this red dot is important — if the batter can see this red dot, then he will know the pitch is a slider and he can better predict its trajectory.

Fig. 7. The batter's view of a slider thrown by a right-handed pitcher: the ball is coming out of the page. The red dot alerts the batter that the pitch is a slider. Copyright ©, 2004, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

4. Magnitude of Forces Acting on a Spinning Ball in Flight
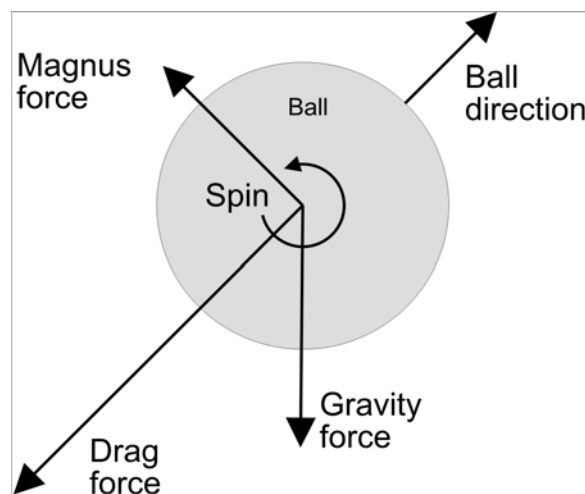
Fig. 8. The forces acting on a spinning ball flying through the air. Copyright ©, 2007, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

Our tactics are to use baseball units (e. g. feet, mph and pounds) for inputs, SI units (e. g. meters, kilograms and seconds) for computations, and baseball units for outputs.

Three forces affect the ball in flight, as shown in Figure 8: gravity pulls the ball downward, air resistance or drag operates in the opposite direction of the ball's motion and, if it is spinning, there is a force perpendicular to the direction of motion. The force of gravity is downward, $F_{\text{gravity}} = m_{ball}g$, where m_{ball} is the mass of the ball and g is the acceleration due to gravity: the magnitude of F_{gravity} is the ball's weight, as in Table 1a.

The magnitude of the drag force opposite to the direction of flight is

$$F_{\text{drag}} = 0.5\rho\pi r_{\text{ball}}^2 C_{\text{d}} v_{\text{ball}}^2 \tag{1}$$

where ρ is air mass density, v_{ball} is the ball speed and r_{ball} is the radius of the ball [Watts and Bahill, 2000, p. 161]. Typical values for these parameters are given in Table 1. Of course SI units should be used in this equation, but if English units are to be used in Equations (1-7) then ρ should be in lb-s^2/ft^4, v_{ball} should be in ft/s, r_{ball} should be in ft, F_{drag} should be in lb and in later equations ω should be in radians/s. For the aerodynamic drag coefficient, C_d, a value of 0.5 is used [Watts and Bahill, 2000, p. 157]: C_d has no units. This drag coefficient is discussed in the *8 Modeling Philosophy* section of this paper.

Table 1a: Typical baseball and softball parameters for line drives [Bahill and Baldwin, 2007]

	Major League Baseball	Little League	NCAA Softball
Ball	Baseball	Baseball	Softball
Ball weight (oz)	5.125	5.125	6.75
Ball weight, $F_{gravity}$, (lb)	0.32	0.32	0.42
Ball radius (in)	1.45	1.45	1.9
Ball radius, r_{ball} (ft)	0.12	0.12	0.16
Pitch speed (mph)	85	50	65
Pitch speed, v_{ball} (ft/s)	125	73	95
Distance from front of rubber to tip of plate (ft)	60.5	46	43
Pitcher's release point: (distance from tip of plate, height), (ft)	(55.5, 6)	(42.5, 5)	(40.5, 2.5)
Bat-ball collision point: (distance from tip of plate, height), (ft)	(3, 3)	(3, 3)	(3, 3)
Bat type	Wooden C243	Aluminum	Aluminum
Typical bat weight (oz)	32	23	25
Maximum bat radius (in)	1.375	1.125	1.125
Speed of sweet spot (mph)	60	45	50
Coefficient of restitution (*CoR*)	0.54	0.53	0.52
Backspin of batted ball (rps)	10 to 70	10 to 70	10 to 70
Backspin of batted ball, ω (rad/s)	63 to 440	63 to 440	63 to 440
Initial batted-ball speed, v_{ball} (ft/s)	135	109	109
Desired ground contact point from the plate (ft)	120 to 240	80 to 140	80 to 150
Air weight density, (lb$_m$/ft^3)	0.065	0.065	0.065
Air mass density, ρ (lb-s^2/ft^4)	0.002	0.002	0.002
Air density depends on altitude, temperature, barometric pressure and humidity.			

NCAA stands for the National Collegiate Athletic Association, which is the governing body for university sports in the United States.

Table 1b: Typical baseball and softball parameters for line drives (SI units)

	Major League baseball	Little League	NCAA Softball
Bat type	Wooden C243	Aluminum	Aluminum
Ball type	Baseball	Baseball	Softball
Pitch speed, v_{ball} (m/s)	38	22	29
Speed of sweet spot (m/s)	27	20	22
CoR	0.54	0.53	0.52
Typical bat mass (kg)	0.9	0.6	0.7
Ball mass, m_{ball} (kg)	0.145	0.145	0.191
Maximum bat radius (m)	0.035	0.029	0.029

Ball radius, r_{ball} (m)	0.037	0.037	0.048
Distance from front of rubber to tip of plate (m)	18.4	14.0	13.1
Pitcher's release point: distance from tip of plate and height	17 m out 2 m up	13 m out 1.5 m up	12 m out 0.8 m up
Bat-ball collision point: distance from tip of plate and height	1 m out 1 m up	1 m out 1 m up	1 m out 1 m up
Backspin of batted ball, ω (rad/s)	100 to 500	100 to 500	100 to 500
Initial batted-ball speed, v_{ball} (m/s)	41	33	33
Desired ground contact point: distance from the plate (m)	37 to 73	24 to 43	24 to 46
Air density, ρ (kg/m^3) This is the average air density for a game played in a major league stadium on a July afternoon.	1.045	1.045	1.045
Air density depends on altitude, temperature, barometric pressure and humidity.			

Table 2 shows typical parameters for major league pitches. We estimate that 90% of major league pitches fall into these ranges. The pitch speed is the speed at the release point: the ball will be going 10% slower when it crosses the plate. In this paper, the equations are general and should apply to many types of spinning balls. However, whenever specific numerical values are given, they are for major league baseball (unless otherwise stated).

Table 2: Typical values for major league pitches [Watts and Bahill, 2000]

Type of pitch	Initial Speed (mph)	Initial Speed (m/s)	Spin rate (rpm)	Spin rate (revolutions per second)	Rotations between pitcher's release and the point of bat-ball contact
Fastball	85 to 95	38 to 42	1200	20	8
Slider	80 to 85	36 to 38	1400	23	10
Curveball	70 to 80	31 to 36	2000	33	17
Change-up	60 to 70	27 to 31	400	7	4
Knuckle ball	60 to 70	27 to 31	30	½	¼

The earliest empirical equation for the transverse force on a spinning object moving in a fluid is the Kutta-Joukowski Lift Theorem

$$\boldsymbol{L} = +\rho \boldsymbol{U} \times \boldsymbol{\Gamma} \tag{2}$$

where $\boldsymbol{L}$ is the lift force per unit length of cylinder, ρ is the fluid density, $\boldsymbol{U}$ is the fluid velocity and $\boldsymbol{\Gamma}$ is the circulation around the cylinder. $\boldsymbol{L}, \boldsymbol{U}$ and $\boldsymbol{\Gamma}$ are vectors. To see the original Sikorsky and Lightfoot 1949 data about C_{lift} and circulation see Alaways [2008]. When this equation is tailored for a baseball [Watts and Bahill, 2000, pp. 77-81], it produces the following equation for the magnitude of the spin-induced force acting perpendicular to the direction of flight.

$$F_{perpendicular} = F_{Magnus} = 0.5 \rho \pi r_{ball}^3 \omega v_{ball} \tag{3}$$

where ω is the spin rate in radians/sec. This is usually called the Magnus force. This is an experimental, not a theoretical equation. If we make the approximation $C_{lift} = \dfrac{r_{ball} \omega}{v_{ball}}$ then Equations (1) and (3) are equivalent to Equations (2) and (3) in McBeath, Nathan, Bahill and Baldwin [2008] and the equations on pages 159-169 of Watts and Bahill [2000].

To show how Equations (1) and (3) work, let us now present a simple numerical example. Assume a 95 mph (42.5 m/s) fastball that has 20 revolutions per second of pure backspin. Using English units and Table 1a, produces

$$F_{\text{drag}} = 0.5\rho\pi r_{\text{ball}}^2 C_{\text{d}} v_{\text{ball}}^2$$

$$F_{\text{drag}} = (0.5)(0.002)(3.14)(0.12)^2(0.5)(139)^2 = 0.44 \text{ lb}$$

Near the beginning of the pitch, the Magnus force will be straight up in the air, that is, pure lift.

$$F_{\text{Magnus}} = 0.5\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball}}$$

$$F_{\text{Magnus}} = (0.5)(0.002)(3.14)(0.12)^3(126)(139) = 0.095 \text{ lb}$$

For this fastball, the Magnus force is about one-third the force of gravity given in Table 1a (0.32). This is consistent with Table 3a.

Table 3a: Gravity-induced and spin-induced drop (with English units) [Bahill and Baldwin, 2007]

Pitch speed and type	Spin rate (rpm)	Duration of flight (msec)	Drop due to gravity (ft)	Spin-induced vertical drop (ft)	Total drop (ft)
95 mph fastball	-1200	404	2.63	-0.91	1.72
90 mph fastball	-1200	426	2.92	-0.98	1.94
85 mph slider	+1400	452	3.29	+0.74	4.03
80 mph curveball	+2000	480	3.71	+1.40	5.11
75 mph curveball	+2000	513	4.24	+1.46	5.70

Table 3b: Gravity-induced and spin-induced drop (with SI units)

Pitch speed and type	Spin rate (rad/s)	Duration of flight (msec)	Drop due to gravity (m)	Spin-induced vertical drop (m)	Total drop (m)
42.5 m/s fastball	-126	404	0.80	-0.28	0.52
40.2 m/s fastball	-126	426	0.89	-0.30	0.59
38.0 m/s slider	+147	452	0.95	+0.23	1.23
35.8 m/s curveball	+209	480	1.13	+0.43	1.56
33.5 m/s curveball	+209	513	1.29	+0.45	1.74

Using SI units and Table 1b, produces

$$F_{\text{drag}} = 0.5\rho\pi r_{\text{ball}}^2 C_{\text{d}} v_{\text{ball}}^2$$

$$F_{\text{drag}} = (0.5)(1.045)(3.14)(0.037)^2(0.5)(42.5)^2 = 2.03 \text{ N}$$

and

$$F_{\text{Magnus}} = 0.5\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball}}$$

$$F_{\text{Magnus}} = (0.5)(1.045)(3.14)(0.037)^3(126)(42.5) = 0.45 \text{ N}$$

For this fastball, the Magnus force is about one-third the force of gravity, which is

$$F_{\text{gravity}} = m_{ball}\text{g} = 0.145 \times 9.8 = 1.42 \text{ N}$$

When the ball's spin axis is not horizontal, the Magnus force should be decomposed into a force lifting the ball up and a lateral force pushing it sideways.

$$F_{\text{upward}} = 0.5\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball}} \sin VaSa \tag{4}$$

where *VaSa* is the angle between the vertical axis and the spin axis (Figure 5). The magnitude of the lateral force is

$$F_{\text{sideways}} = 0.5\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball}} \cos VaSa \tag{5}$$

Finally, if the spin axis is not perpendicular to the direction of motion (as in the case of the slider), the magnitude of the cross product of these two vectors will depend on the angle between the spin axis and direction of motion, this angle is called *SaD* (Figure 9). In aeronautics, it is called the angle of attack.

$$F_{\text{lift}} = 0.5\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball}} \sin VaSa \sin SaD \tag{6}$$

$$F_{\text{lateral}} = 0.5\rho\pi r_{\text{ball}}^3 \omega v_{\text{ball}} \cos VaSa \sin SaD \tag{7}$$
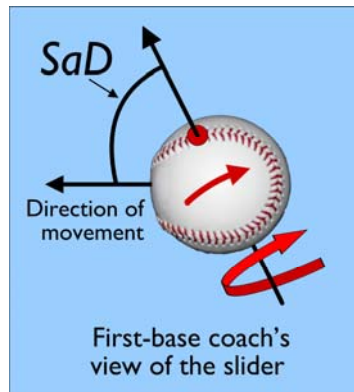

Fig. 9. The first-base coach's view of a slider thrown by a right-handed pitcher. This illustrates the definition of the angle *SaD*. Copyright ©, 2007, Bahill, from http://www.sie.arizona.edu/sysengr/slides/ used with permission.

The spin-induced force on the ball changes during the pitch. Its magnitude decreases, because the drag force slows the ball down by about 10%. Its direction changes, because gravity is continuously pulling the ball downward, which changes the direction of motion of the ball by 5 to 10 degrees. However, the ball acts like a gyroscope, so the spin axis does not change. This means that, for a slider, the angle *SaD* increases and partially compensates for the drop in speed in Equations (6) and (7).

The right-hand rules for the lateral deflection of a spinning ball and Equations (1) to (7) apply to pitched and also batted-balls, except it is harder to make predictions about the magnitude of deflection of batted-balls, because the data about the spin of batted-balls are poor. The right-hand rules and these equations can also be applied to soccer, tennis and golf, where speeds, spins and deflections are similar to baseball. However, the right-hand rules and these equations would be inappropriate for American football, because the spin axis of a football is almost coincident with the direction of motion. Therefore the angle *SaD* is near zero and consequently the spin-induced deflections of a football are small [Rae, 2004].

5. Effects of Air Density on a Spinning Ball in Flight

The distance a ball travels is inversely related the air density. But the explanation for this is not straightforward. Equations 1 and 3 show that both the drag and Magnus forces are directly proportional to the air density. So if air density gets smaller, the drag force gets smaller, this allows the ball to go farther. But at the same time, as air density gets smaller, the Magnus force also gets smaller, which means that the ball will not be held aloft as long and will therefore not go as far. So these two effects are in opposite directions. We have built a computer simulation that implements the above equations. This simulation shows that the change in the drag force has a greater affect on the trajectory of the ball than the change in the Magnus force does; therefore, as air density goes down, the range of a potential home run ball increases. A ten-percent decrease in air density produces a four-percent increase in the distance of a home run ball: however, the increase is less than this for pop-ups and greater than this for line drives.

Air density is inversely related to altitude, temperature and humidity, and is directly related to barometric air pressure. These relationships are given in Equation 8. While they appear nonlinear, we will obtain a linear approximation that is accurate over the range of values observed at major league ballparks. An equation from

the WeatherLink Software [2009; CRC Handbook of Chemistry & Physics, 1980-81)] for these relationships is

$$Air\ Density = \rho = 1.2929 \times \frac{273}{Temp+273} \times \frac{Air\ Pres - (SVP \times RH/100)}{760} \tag{8}$$

where *Air Density* is in kg/m^3.

Temp is temperature in degrees Celsius.

SVP is saturation vapor pressure in mm Hg.

RH is relative humidity as a percentage.

Air Pres is the pressure of the air in mm of Hg.

This equation uses the absolute (or actual) atmospheric air pressure, which is called *station pressure* because it is the air pressure at a particular weather station. It can be computed from the U. S. Weather Service sea-level corrected barometric pressure with the following formula.

$$Air\ Press = Barometric\ Pressure\,[\,e^{\frac{-gM\ Altitude}{R\,(Temp+273.15)}}\,] \tag{9}$$

where g is the Earth's gravitational acceleration (9.80665 m/s^2 at sea level),

M is the molecular mass of air (0.0289644 kg/mole),

R is the Universal Gas Constant (8.31447 joules/ °K mole),

Altitude is the altitude of the ballpark in meters,

and *Temp* is the temperature in °C.

But what is *Temp* the temperature of? As a simple approximation in the following examples, we have used the temperature of the baseball stadium. But the above equation should be integrated with respect to the time-averaged temperature from the baseball stadium to mean sea level. Because this is impossible, the National Weather Service [2001] uses nine different approximations: about them they write, "There is no single true, correct solution of Sea Level Pressure … only estimates." For any given time and place the most accurate measure of air pressure for Equation 8 would be a local barometer that is not corrected to sea level (i. e. with its altitude set to 0).

Dozens of equations have been fit to the experimental saturation vapor pressure (*SVP*) data. Here is one by Buck [1996].

$$SVP = 4.5841\,e^{\frac{\left(18.687-\frac{Temp}{234.5}\right)*Temp}{257.14+Temp}} \tag{10}$$

As before*, Temp* is in degrees Celsius and *SVP* is in mm Hg.

Air density is inversely related to altitude, temperature and humidity, and is directly related to barometric pressure. For the range of values in major league ballparks, the altitude is the most important of the four input parameters. Table 4 gives values for a typical late-afternoon summer game, assuming that the stadium roofs are open and there are no storms. For these examples, baseball units are used instead of SI units.

Table 4: Air density in some typical baseball stadiums

	Altitude (feet above sea level)	Temperature (degrees Fahrenheit) average daily maximum in July	Relative humidity, on an average July afternoon	Average barometric pressure in July (inch of Hg)	Air density (kg/m^3)
Denver	5190	88	34 %	29.98	0.96
Houston	45	94	63 %	29.97	1.11
Minneapolis	815	83	59 %	29.96	1.11
Phoenix	1086	104	20 %	29.81	1.07
San Francisco	0	68	65 %	29.99	1.19
Seattle	10	75	49 %	30.04	1.18

Weather data such as these can be obtained from http://www.weather.com and

http://www.wunderground.com/. The multi-year average July afternoon relative humidity data came from http://www.ncdc.noaa.gov/oa/climate/online/ccd/avgrh.html. Average daily maximum temperature and average afternoon humidity are also available at www.weatherReports.com. The multi-year average July barometric pressure data came from presave.ct located at http://eande.lbl.gov/IEP/high-radon/data/lbnl-met.html. Estimates of barometric pressure are also available at http://www.usairnet.com/weather/maps/current/barometric-pressure/. The multi-year average July maximum daily temperatures came from http://hurricane.ncdc.noaa.gov/cgi-bin/climatenormals/climatenormals.pl?directive=prod_select2&prodtype=CLIM81&subrnum=. Programs that calculate air density can be downloaded from Linric Company (http://www.linric.com/) or they can be used on-line at http://www.uigi.com/WebPsycH.html.

For a potential home run ball, both the drag and the lift (Magnus) forces are the greatest in San Francisco, where the park is just at sea level, and smallest in the "mile high" city of Denver. However, as previously stated, the drag force is more important than the Magnus force. Therefore, if all collision parameters (e.g. pitch speed, bat speed, collision point, etc.) are equal, a potential home run will travel the farthest in Denver and the shortest in San Francisco.

Table 5: Values used in the simulations

	Altitude (feet above sea level)	Temperature (degrees Fahrenheit)	Relative Humidity (percent)	Barometric pressure (inch Hg)	Air density (kg/m^3)	Air density, percent change from mean
Low altitude	0	85	50	29.92	1.14	9.4
Low temperature	2600	70	50	29.92	1.08	3.3
Low humidity	2600	85	10	29.92	1.06	1.6
Low barometric pressure	2600	85	50	29.33	1.02	-2.2
Mid-level	2600	85	50	29.92	1.04	-0.2
High barometric pressure	2600	85	50	30.51	1.06	1.8
High humidity	2600	85	90	29.92	1.02	-2.0
High temperature	2600	100	50	29.92	1.00	-3.9
High altitude	5200	85	50	29.92	0.95	-8.9
Highest density	0	70	10	30.51	1.22	16.8
Lowest density	5200	100	90	29.33	0.87	-16.8

These values were chosen to show realistic numbers with natural variation. On any given afternoon in July, it is almost certain that baseball games will be played at the high and low ends of all these ranges.

To understand how the four fundamental variables, altitude, temperature, humidity and barometric pressure, determine the air density, these equations were evaluated at eighty-one experimental points. These points were selected at the low, middle and high values of the fundamental variables, or at 3^4 or 81 points. An edited regression output is given in Table 6.

Surprisingly, a simple linear equation explains most of the changes, or variability, in the air density values. The linear algebraic equation for air density obtained by least squares analysis is

Δ *Air density* (percent change from mean level) =
- 0.0035 (*Altitude* - 2600)
- 0.2422 (*Temperature* - 85)
- 0.0480 (*Relative Humidity* - 50)
+ 3.4223 (*Barometric Pressure* - 29.92)   (11)

where Δ *Air density* is stated as a percent change from mean level of 1.045, *Altitude* is in feet, *Temperature* is in degrees Fahrenheit, *Relative Humidity* is in percent and *Barometric Pressure* is in inches of Hg. The parameter estimates are taken from Table 6. This equation can be re-expressed to give the air density in kg/m^3

Air density = ρ = 1.045 + 0.01045{
- 0.0035 (*Altitude* - 2600)

$$- 0.2422 \ (Temperature - 85)$$
$$- 0.0480 \ (Relative \ Humidity - 50)$$
$$+ 3.4223 \ (Barometric \ Pressure - 29.92)\}$$ (12)

This *Air density* is ρ in Table 1b and Equations (1) to (8).

Note that the factors are in different dimensions with different ranges. Hence, the magnitudes of the coefficients should be interpreted in this light. That is, a coefficient with a larger magnitude does not necessarily mean it has a greater impact on the response. Also, keep in mind that the equations that yield the air density values are deterministic. That is, there is no random variation. Hence, the sum of squares residual is the variation remaining after predicting the response from the linear approximation. There is no pure error, but rather simply lack of fit to the true model. The least squares analysis differentiates between the variables for the range of the eighty-one observations as follows. Altitude explains 80% of the variability; temperature explains 13%, barometric pressure accounts for 4% and relative humidity accounts for 3%.

Table 6: Edited Regression Summary for Linear Approximation (JMP and Excel)

Summary of Fit	
RSquare	0.993
RSquare Adjusted	0.993
Root Mean Square Error	0.71
Observations (or Sum Weights)	81

Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	5662	1415	2783
Error	76	39	0.51	
C. Total	80	5701		

Parameter Estimates			
Term	Estimate	Std Error	t Ratio
Intercept	0.0		
Altitude (ft) - 2600	-0.0035	0.0000	-94
Temperature (F) - 85	-0.2422	0.0065	-37
Relative Humidity (%) - 50	-0.0480	0.0024	-20
Sea level Corrected Barometric Pressure (inch Hg) - 29.92	3.4223	0.1643	21

Since Equation (11) is linear, the impact of each factor can be shown graphically. Figure 10 shows the changes in air density that should be expected over the range of parameter values that would be typical for a baseball stadium on an afternoon in July in the United States of America. It shows that altitude is the most important factor, followed by temperature, barometric pressure and relative humidity. Since the factor ranges given are indicative of their natural variation, larger absolute slopes means stronger effects. These results are for baseball and should not be used for other purposes, such as calculating safe takeoff parameters for a small airplane.

The linear Equation (11) explains 99.3 percent of the variation of air density across the 81 setting. However, the unexplained variation, as given by the prediction standard error is 0.71, suggesting that further improvement is possible. (It is possible to obtain a very high R2 and still have unexplained variability.) Figure 10 shows a quadratic pattern between the residuals and the predicted values of the linear approximation, suggesting that second order terms might be helpful. Since altitude is the most important factor the square of its value is a likely candidate. After fitting a regression to the complete quadratic model, that also includes four pure square terms and six cross product terms, the conjecture is confirmed, the square of altitude does play a role. In addition, the cross product term between altitude and temperature, is even more important, although they are a magnitude smaller than the linear altitude and temperature terms in their effect.
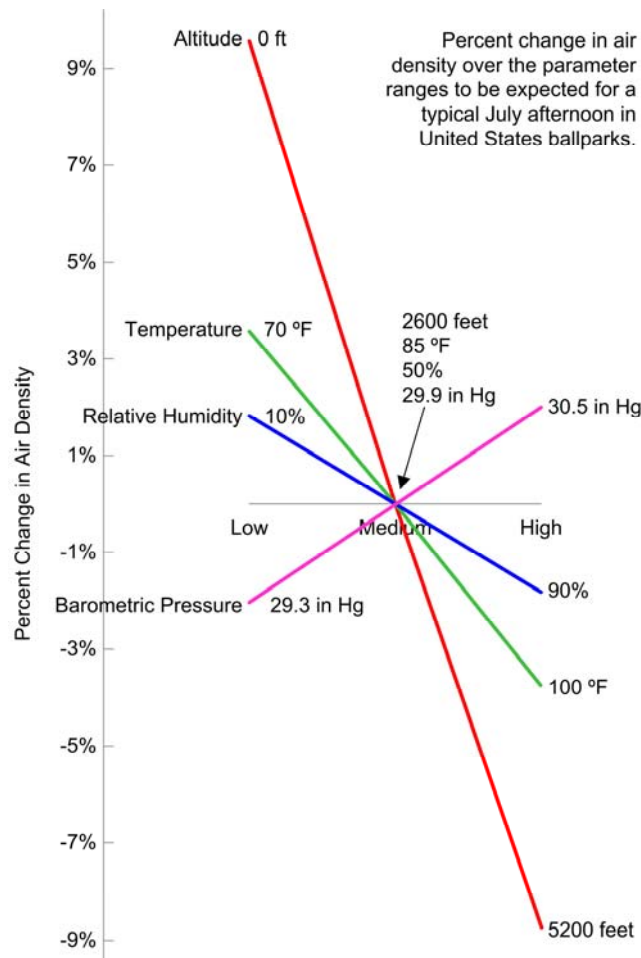
Fig. 10. Air density depends on altitude, temperature, barometric pressure and relative humidity.
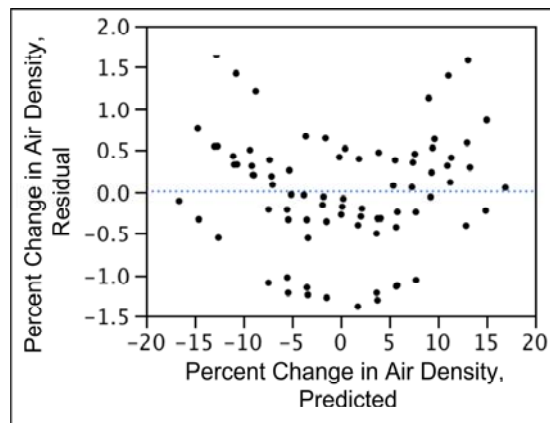

Fig. 11. Residuals versus predicted air density for the linear approximation

The impact of augmenting the model with these two second order terms raises the percentage of variation explained only slightly (from 99.3 to 99.5%), but decreases the unexplained variation, as measured by the prediction standard error from 0.71 to 0.61. The corresponding model is given by Equation (13).

Δ *Air density* (percent change from mean level) =

- 0.0035 (*Altitude* - 2600)
- 0.2422 (*Temperature* - 85)
- 0.0480 (*Relative Humidity* - 50)
+ 3.4223 (*Barometric Pressure* - 29.92)
+ 0.000000061 $\{(Altitude - 2600)^2 - 4506667\}$
+ 0.000012 (*Altitude* - 2600)•(*Temperature* -85)   (1 3)

Please note that this is not a traditional sensitivity analysis. In a sensitivity analysis each parameter would be changed by a certain percent and then the resulting changes in the output would be calculated [Smith, Szidarovszky, Karnavas and Bahill, 2008]. For baseball, if we change each parameter by 5% we find that the semirelative sensitivity of air density with respect to barometric pressure is 1.07: the semirelative sensitivity of air density with respect to temperature is -0.21: the semirelative sensitivity of air density with respect to altitude is -0.1: and the semirelative sensitivity of air density with respect to relative humidity is -0.02. The reason for the different results is that the high, medium and low barometric pressures that could be expected on a July afternoon in a major league baseball stadium are 775, 760 and 745 mm Hg. These changes are much less than 5%. Whereas, the high, medium and low altitudes that could be expected in a major league baseball stadium are 5200, 2600 and 0 feet. These changes are much more than 5%. Stated simply, there would be a greater change in air density due to moving from San Francisco to Denver, than there would be due to moving from fair weather to stormy weather.

The *range* of a batted ball is defined as the distance from home plate to the spot where it first hits the ground. Table 7 shows the range for a perfectly hit simulated baseball. The pitch is an 85 mph (38 m/s) fastball with 1200 rpm backspin, the ball hits the sweet spot of the bat, which is going 55 mph (24.6 m/s), *CoR* is 0.55, the ball is launched upward at 34 degrees with 2000 rpm of backspin. The ball is in the air for five and a half seconds. This is a potential home run ball. Reducing the air density by 10% from 1.0 to 0.9 increased the range of this potential home run ball by four percent.

Table 7: Range as a function of air density

Air Density (kg/m^3)	Range (ft)			Range (m)		
	Home run	Pop up	Line drive	Home run	Pop up	Line drive
1.3	351	205	114	107	62	35
1.2	363	209	129	111	64	39
1.1	377	214	142	115	65	42
1.0	392	219	158	119	67	48
0.9	408	224	176	124	68	54
0.8	425	230	196	130	70	60

The pop-up was launched at 82 mph (36.7 m/s) at an upward angle of 58 degrees with a backspin of 4924 rpm. The line drive was launched at 92 mph (41.1 m/s) at an upward angle of 15 degrees with a backspin of 263 rpm.

In this section, average values were used. Of course, ball games are not played at the average values and the actual values are not constant throughout the game. In particular, wind speed and direction could change on a minute-by-minute basis. In this section, the effects of prevailing winds or height and distance of the outfield walls were not modeled. Chambers, Page, and Zaidins, [2003] have written that for most games played at Coors Field in Denver there was a light breeze (e. g. 5 mph, 2.2 m/s) blowing from center field toward home plate. They stated that the outfield walls at Coors Field were farther back than in most stadiums. They concluded that these two factors reduced the number of home runs by three to four percent, which nearly compensated for Denver's high altitude.

Greg Rybarczyk's data (personal communication, 2009) show that the greatest wind effects in major league stadiums are in San Francisco's AT&T Park where the average is a gentle breeze blowing from home plate into the right-center field stands at 10 mph (4.5 m/s).

6. Vertical Deflections of Specific Pitches

The magnitude of the gravity and spin-induced drops for three kinds of pitches at various speeds (as determined by our simulations) are shown in Table 3. Our baseball simulations include the force of gravity, the drag force, and the vertical and horizontal spin-induced forces [Bahill and Karnavas, 1993; Watts and Bahill, 2000; Bahill and Baldwin, 2004]. Looking at one particular row of Table 3, a 90 mph (40.2 m/s) fastball is in the air for 426 msec, so it drops 2.92 feet (0.89 m) due to gravity ($\frac{1}{2} gt^2$, where the gravitational constant g is 32.2 ft/sec^2 or 9.8 m/sec^2 and *t* is the time from release until the point of bat-ball collision). But the backspin lifts this pitch 0.98 ft (0.3 m), producing a total drop of 1.94 ft (0.59 m) as shown in Table 3. In the spin rate column, negative numbers are backspin and positive numbers are top spin. In the spin-induced

vertical drop column, negative numbers mean the ball is being lifted up by the Magnus force. All of the pitches in Table 3 were launched horizontally - that is, with a launch angle of zero: that is why they are different from the pitches in Figures 1 and 2. The angle *VaSa* was also set to zero (simulating an overhand delivery): therefore pitches thrown with a three-quarter arm delivery would have smaller spin-induced deflections than given in Table 3.

A batter's failure to hit safely is most likely caused by his fallibility in predicting where and when the ball will reach the bat-ball contact point. Vertical misjudgment of this potential bat-ball contact point is the most common cause of batters' failure [Bahill, and Baldwin, 2003; Baldwin and Bahill, 2004]. The vertical differences between the curveballs and fastballs in Table 3 are greater than three feet (1 m), whereas the difference between the two speeds of fastballs is around three inches (7 cm) and the difference between the two speeds of curveballs is around seven inches (18 cm). However, the batter is more likely to make a vertical error because speed has been misjudged than because the kind of pitch has been misjudged [Bahill, and Baldwin, 2003; Baldwin and Bahill, 2004]. A vertical error of as little as one-third of an inch (8 mm) in the batter's swing will generally result in a failure to hit safely [Bahill, and Baldwin, 2003; Baldwin and Bahill, 2004].

The spin on the pitch causes both vertical and horizontal deflections of the ball's path. When a batter is *deciding* whether to swing, the horizontal deflection is more important than the vertical, because the umpire's judgment with respect to the corners of the plate has more precision than his or her judgment regarding the top and bottom of the strike zone. However, after the batter has decided to swing and is trying to *track and hit* the ball, the vertical deflection becomes more important, because of the dimensions of the sweet spot.

7. Effects of Air Density on Specific Pitches

A reduction in air density would also reduce the drag and the Magnus forces on the pitch. Table 8 shows the speed and the height of the ball when it crosses the front edge of the plate for a 90 mph (40.2 m/s) fastball launched horizontally with 1200 rpm of backspin using an over arm delivery and for a 75 mph (33.5 m/s) curveball launched upward at two degrees with 2000 rpm of pure top spin.

Table 8: Pitch variations with air density

Air Density (kg/m^3)	90 mph fastball		75 mph curveball	
	Speed (mph)	Height (ft)	Speed (mph)	Height (ft)
1.3	77.52	4.02	62.22	1.54
1.2	78.45	3.96	65.94	1.75
1.1	79.38	3.90	66.68	1.95
1.0	80.33	3.85	67.42	2.14
0.9	81.29	3.79	68.17	2.33
0.8	82.26	3.74	68.93	2.53

A ten-percent decrease in air density from 1.1 to 1.0 increases the speed of the fastball by one percent and reduces its rise by four percent. A ten-percent decrease in air density from 1.1 to 1.0 increases the speed of the curveball by one percent and reduces its drop by nine percent. Earlier in this paper we wrote, if all other things were equal, a ten-percent decrease in air density would produce a four-percent increase in the distance of a home run ball. Well now it can be seen that all other things will not be equal: the pitch speed will be larger (the bat speed will not change). Using the higher pitch speed changes the conclusion about the range of the batted ball by only one hundredth of a percent.

8. Modeling Philosophy

A model is a simplified representation of a particular view of a real system. No model perfectly matches all views of its real system. If it did, then there would be no advantage to using the model. Although the equations and numerical values in this paper might imply great confidence and precision in our numbers, it is important to note that our equations are only models. The Kutta-Joukowski lift equation and subsequent derivations are not theoretical equations, they are only approximations fit to experimental data. More complicated equations for the forces on a baseball have been used (e. g. see [Frohlich, 1984; Adair, 2002 and

2004; Sawicki, Hubbard and Stronge, 2003 and 2004; Nathan, et al., 2006]). Furthermore, our model only considered certain aspects of the baseball in flight. We ignored the possibility that air flowing around certain areas of the ball might change from turbulent to laminar flow en route to the plate. Our equations did not include effects of shifting the wake of turbulent air behind the ball. En route to the plate, the ball loses 10% of its linear velocity [Watts and Bahill, 2000] and 2% of its angular velocity [McBeath, Nathan, Bahill and Baldwin, 2008]: we did not include this reduction in angular velocity in our simulation. We ignored the difference between the center of mass and the geometrical center of the baseball [Briggs, 1959]. We ignored possible differences in the moments of inertia of different balls. We ignored the precession of the spin axis. In computing velocities due to bat-ball collisions, we ignored deformation of the ball and energy dissipated when the ball slips across the bat surface. Finally, as we have already stated, we treated the drag coefficient as a constant.

In our models, we used a value of 0.5 for the drag coefficient, C_d. However, for speeds over 80 mph (36 m/s) this drag coefficient may be smaller [Watts and Bahill, 2000, p. 157; Frohlich, 1984; Adair, 2002; Sawicki, Hubbard and Stronge, 2004]. There are no wind-tunnel data showing the drag coefficient of a spinning baseball over the range of velocities and spin rates that characterize major league pitches [Sawicki, Hubbard and Stronge, 2004]. Data taken from a half-dozen studies of spinning baseballs, nonspinning baseballs and other balls showed C_d between 0.15 and 0.5 [Sawicki, Hubbard and Stronge, 2003]. In most of these studies, the value of C_d depended on the speed of the airflow. In the data of [Nathan, et al., 2006], the drag coefficient can be fit with a straight line of $C_d = 0.45$, although there is considerable scatter in these data. The drag force causes the ball to lose about 10% of its speed en route to the plate. The simulations of Alaways, Mish and Hubbard [2001] also studied this loss in speed. Data shown in their Figure 9 for the speed lost en route to the plate can be nicely fitted with $PercentSpeedLost = 20C_d$, which implies $C_d = 0.5$.

Our models are also hampered by limited data for the spin of the ball. The best, published experimental data for the spin rate of different pitched baseballs comes from Selin's cinematic measurements of baseball pitches [Selin, 1959]. Furthermore, there are no experimental data for the spin on the batted ball. Table 2 summarizes the best estimates of speed and spin rates for the most popular major league pitches.

The numerical values used for the parameters in our equations have uncertainty. However, the predictions of the equations match baseball trajectories quite well. When better experimental data become available for parameters such as C_d and spin rate, then values of other parameters will have to be adjusted to maintain the match between the equations and actual baseball trajectories.

There are many models for the flight of the baseball. The models of Bahill [Bahill, and Baldwin, 2007; Watts and Bahill, 2000]; Adair [2002 and 2004], Nathan [Nathan, 2006; McBeath, Nathan, Bahill and Baldwin, 2008] and Hubbard [Sawicki, Hubbard, and Stronge, 2003 and 2004] all give different absolute numerical results. But, we believe, all of them will give the same comparative results. Meaning that they all should show that a ten-percent decrease in air density produces about a four-percent increase in the distance of a home run ball with the increase being less for pop-ups and greater for line drives.

The importance of this present paper lies in comparisons rather than absolute numbers. Our model emphasizes that the right-hand rules show the direction of forces acting on a spinning ball in flight. The model provides predictive power and comparative evaluations of behavior of different types of pitches.

Larry Stark [1968] explained that models are ephemeral: they are created, they explain a phenomenon, they stimulate discussion, they foment alternatives and then they are replaced by new models. When there are better wind-tunnel data for the forces on a spinning baseball, then our equations for the lift and drag forces on a baseball might be supplanted by newer parameters and equations. But we think our models, based on the right hand rules showing the direction of the spin-induced deflections, will have permanence: they are not likely to be superseded.

9. Summary

Air density is inversely related to altitude, temperature and humidity, and is directly related to barometric pressure, Equation (12). Both the drag force (Equation 1) and the Magnus force (Equation 3) are directly proportional to the air density. So if air density gets smaller, the drag force gets smaller, this allows the ball

to go farther: But at the same time, as air density gets smaller, the Magnus force also gets smaller, which means that the ball will not be held aloft as long and will therefore not go as far. These two effects are in opposite directions. Simulation shows that the change in the drag force affects the trajectory of the ball more than the change in the Magnus force. Therefore, as air density goes down, the range of a potential home run ball increases. On a typical July afternoon in a major league baseball stadium, a ten-percent decrease in air density can produce a four-percent increase in the distance of a home run ball.

10. Acknowledgements

11. References

[1]   R. K. Adair. Comments on "How to hit home runs: Optimum baseball bat swing parameters for maximum range trajectories," by G. S. Sawicki, M. Hubbard and W. J. Stronge. *Am. J. Phy.* 2004, **73** (2): 184-185.

[2]   R. K. Adair. *The Physics of Baseball.* New York: HarperCollins, 2002.

[3]   L. W. Alaways. Aerodynamics of a curve-ball: the Sikorsky/Lightfoot lift data. *Proceedings of 7th ISEA Conference.* Biarritz, June 2-6, 2008.

[4]   L. W. Alaways, S. P. Mish and M. Hubbard. Identification of release conditions and aerodynamic forces in pitched-baseball trajectories. *Journal of Applied Biomechanics.* 2001, **17**: 63-76.

[5]   A. T. Bahill and T. LaRitz. Why can't batters keep their eyes on the ball? *American Scientist.* 1984, **72**: 249-253.

[6]   A. T. Bahill and D. G. Baldwin. Mechanics of baseball pitching and batting, Chapter 16. *Applied Biomedical Engineering Mechanics, (Ed. Dhanjoo Ghista).* CRC Press and Taylor & Francis Asia Pacific. 2008, pp. 445-488.

[7]   A. T. Bahill and D. G. Baldwin. The rising fastball and other perceptual illusions of batters. *Biomedical Engineering Principles in Sports. (Editors G. Hung and J. Pallis).* New York: Kluwer Academic. 2004, pp. 257-287.

[8]   A. T. Bahill and D. G. Baldwin. Describing baseball pitch movement with right-hand rules. *Computers in Biology and Medicine.* 2007, **37**: 1001-1008.

[9]   A. T. Bahill and D. G. Baldwin. The vertical illusions of batters. *Baseball Research Journal.* 2003, **32**: 26-30.

[10] A. T. Bahill and W. J. Karnavas. The ideal baseball bat. *New Scientist.* 1991, **130**(1763): 26-31.

[11] A. T. Bahill and W. J. Karnavas. The perceptual illusion of baseball's rising fastball and breaking curve ball. *Journal of Experimental Psychology: Human Perception and Performance.* 1993, **19**: 3-14.

[12] A. T. Bahill, D. G. Baldwin and J. Venkateswaran. Predicting a baseball's path. *American Scientist.* 2005, **93**(3): 218-225.

[13] A. T. Bahill, R. Botta and J. Daniels. The Zachman framework populated with baseball models. *Journal of Enterprise Architecture.* 2006, **2**(4): 50-68.

[14] A. T. Bahill. The ideal moment of inertia for a baseball or softball bat. *IEEE Trans. Systems, Man, and Cybernetics -- Part A Systems and Humans.* 2004, **34** (2): 197-204.

[15] D. G. Baldwin and A. T. Bahill. A model of the bat's vertical sweetness gradient. *The Engineering of Sport* . 5, 2. (Eds.)

[16] M. Hubbard, R. D. Mehta and J. M. Pallis. *Proceedings of the 5th International Engineering of Sport Conference.* 2004, pp. 13-16.

[17] Davis, CA. *International Sports Engineering Association (ISEA).* Sheffield, UK, ISBN 0-9547861-1-4. 2004, 2: . 305-311.

[18] D. G. Baldwin, A. T. Bahill, and A. Nathan. Nickel and dime pitches. *Baseball Research Journal.* 2007, **35**: 25-29.

[19] D. G. Baldwin. Snake Jazz, Xlibris Corp, www.Xlibris.com, 2007.

[20] H. Barkla and L. Auchterlonie. The Magnus or Robins effect on rotating spheres. *Journal of Fluid Mechanics.* 1971, **47**: 437-447.

[21] L. J. Briggs. Effect of spin and speed on the lateral deflection (curve) of a baseball and the Magnus effect for smooth spheres. *American Journal of Physics.* 1959, **27**(8): 589-596.

[22] A. L. Buck. New equations for computing vapor pressure and enhancement factor. *J. Appl. Meteorol.* 1981, **20**: 1527-1532. Buck fine-tuned his equation in 1996.

[23] F. Chambers, B. Page and C. Zaidins. Atmosphere, weather, and baseball: how much farther do baseballs really fly at Denver's Coors Field? *The Professional Geographer.* 2003, **55**(4): 491-508.

[24] *CRC Handbook of Chemistry & Physics, 61st edition.* Boca Raton: CRC Press. 1980-1981, pp. F-10.

[25] C. Frohlich. Aerodynamic drag crisis and its possible effect on the flight of baseballs. *Am. J. Physics*. 1984, **52** (4): 325-334.

[26] NASA. http://www.grc.nasa.gov/WWW/K-12/airplane/bga.html, July 2008, retrieved April 2009.

[27] A. M. Nathan, J. Hopkins, L. Chong and H. Kaczmarski. *The effect of spin on the flight of a baseball.* SABR conference, Seattle, June 2006.

[28] I. Newton. A letter of Mr. Isaac Newton, of the University of Cambridge, containing his new theory about light and colors. *Philosophical Transactions of the Royal Society*. **7** (1671-1672): 3075-3087.

[29] M. K. McBeath, A. M. Nathan, A.T. Bahill and D. G. Baldwin. Paradoxical pop-ups: Why are they difficult to catch? *American Journal of Physics*. 2008, **76**(8): 723-729.

[30] NOAA's National Weather Service. *Surface pressure analysis ... clearing the confusion in the presence of different solutions*, April 2001, http://www.crh.noaa.gov/unr/?n=mslp, accessed April 2009.

[31] W. J. Rae. Mechanics of the forward pass. *Biomedical Engineering Principles in Sports (Editors G. Hung and J. Pallis)*. New York: Kluwer Academic. 2004, pp. 291-319.

[32] B. Robins. New Principles of Gunnery: Containing, the Determination of the Force of Gun-powder, and Investigation of the Difference in the Resisting Power of the Air to Swift and Slow Motions. *J. Nourse*. London, Available online through Early English Books Online, 1742.

[33] G. S. Sawicki, M. Hubbard, and W. J. Stronge. How to hit home runs: Optimum baseball bat swing parameters for maximum range trajectories. *Am. J. Phy*. 2003, **71**(11): 1152-1162.

[34] G. S. Sawicki, M. Hubbard, and W. J. Stronge. Reply to comments on "How to hit home runs: Optimum baseball bat swing parameters for maximum range trajectories." *Am. J. Phy*. 2004, **73**(2): 185-189.

[35] C. Selin. An analysis of the aerodynamics of pitched baseballs. *The Research Quarterly*. 195, **30**(2): 232-240.

[36] E. D. Smith, F. Szidarovszky, W. J. Karnavas and A. T. Bahill. Sensitivity analysis, a powerful system validation technique. The Open Cybernetics and Systemics Journal. http://www.bentham.org/open/tocsj/openaccess2.htm. 2008, **2**: 39-56, doi: 10.2174/1874110X00802010039

[37] L. Stark. *Neurological Control Systems, Studies in Bioengineering.* New York: Plenum Press, 1968.

[38] R. G. Watts and A. T. Bahill. *Keep Your Eye on the Ball: Curve Balls, Knuckleballs and Fallacies of Baseball.* New York: W. H. Freeman, 2000.

[39] *WeatherLink Software*. http://www.davisnet.com/weather/products/software.asp; accessed April 2009.

Author Bios

Terry Bahill is a Professor of Systems Engineering at the University of Arizona in Tucson. He received his Ph.D. in electrical engineering and computer science from the University of California, Berkeley, in 1975. Bahill has worked with BAE Systems in San Diego, Hughes Missile Systems in Tucson, Sandia Laboratories in Albuquerque, Lockheed Martin Tactical Defense Systems in Eagan MN, Boeing Information, Space and Defense Systems in Kent WA, Idaho National Engineering and Environmental Laboratory in Idaho Falls and Raytheon Missile Systems in Tucson. For these companies he presented seminars on Systems Engineering, worked on system development teams and helped them describe their Systems Engineering Process. He holds a U.S. patent for the Bat Chooser™, a system that computes the Ideal Bat Weight™ for individual baseball and softball batters. He received the Sandia National Laboratories Gold President's Quality Award. He is a Fellow of the Institute of Electrical and Electronics Engineers (IEEE), of Raytheon and of the International Council on Systems Engineering (INCOSE). He is the Founding Chair Emeritus of the INCOSE Fellows Selection Committee. His picture is in the Baseball Hall of Fame's exhibition "Baseball as America." You can view this picture at http://www.sie.arizona.edu/sysengr/.

Dave Baldwin, as a relief pitcher for the Washington Senators, Milwaukee Brewers and Chicago White Sox during the 1960s and 1970s, compiled a lifetime ERA of 3.08 with 6 wins, 11 losses and 22 saves in 176 games. Following a 16-year professional baseball career, he earned a Ph.D. in genetics and an M.S. in systems engineering from the University of Arizona. Retired now after careers as a scientist, engineer and artist, Baldwin collaborates on studies of the science of baseball and is a member of the Society for American Baseball Research (SABR). His baseball memoir, *Snake Jazz*, has been published by Xlibris (2007). A description of the book and Baldwin's various careers can be seen at http://www.snakejazz.com.

John Ramberg is Professor Emeritus of Systems and Industrial Engineering at the University of Arizona. He is a Fellow of the American Statistical Association (ASA), the American Society for Quality (ASQ) and the Institute of Industrial Engineers. Dr. Ramberg earned a BS in Electrical Engineering (Industrial Engineering) at the University of Minnesota, an MS in Operations Research and PhD in Engineering Statistics, from Cornell University. He served on the University of Iowa faculty (statistics and

management engineering) from 1967-78, and the University of Arizona faculty from 1978 – 2004. He served as Editor-in-Chief of *Journal of Quality Technology*, and is currently a member of its editorial board. He consults and writes on engineering statistics, and technical and managerial issues underlying improvement schemes such as six sigma.